UNIVERSITÀ DI PARMA

DIPARTIMENTO DI INGEGNERIA E ARCHITETTURA

Course	**Computational Mechanics of Structures** (6 CFU)
Year 1	Master degree in Civil Engineering
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Aims

To present concepts and tools for computational mechanics applied to generic solid structures.

Contents

Basic concepts in computational mechanics.
Introduction to the finite element method: displacement method for plane beam structures.

Introduction to numerical analysis

Numerical solution of nonlinear equations, solution of systems of linear equations, eigenvalues and eigenvectors of a matrix, interpolation of functions, integration (quadrature) of functions, numerical solution of ordinari differential equations (ODEs).

Variational methods

Weak and strong form of a physical problem. Natural and essential boundary conditions. Variational principles. Virtual work theorem. Approximate polynomial solution. Bubnov-Galerkin method.
General formulation of a problem by using finite elements: differential and integral forms.
Minimum potential energy principle. Displacement field approximation. Rayleigh-Ritz method.

Residual methods

Weighted residual method: subdomain method, collocation method, least square method, Galerkin method. The finite element method as a particular case of the Weighted residual method.

Basic concepts of the finite element method

Algebraic static and dynamic equilibrium equations of a structure discretized by finite elements. Stiffness matrix $\mathbf{K}$ and nodal force vector $\mathbf{f}$. Stiffness matrix assembling. Treatment of boundary conditions and their classification: linear and non linear, *single freedom constraints*, *multi freedoms constraints*. *Master-slave* method, *penalty* method, Lagrange's multipliers method.

Structural discretisation with finite elements

Choice of the finite element and of the shape functions. Shape functions in the local reference system and their derivatives. Examples of linear shape functions. Isoparametric elements: convergence requirements. Lagrangian and Serendipity elements.
Isoparametric elements in one, two and three dimensions.
Numerical integration methods. Variable transformation in 1D, 2D, 3D. Gauss rule. Accuracy of the numerical integration. Examples.

Use of finite elements in non linear problems

Eigen analysis: linear buckling problems (geometry stiffness matrix), vibration mode shapes of a structure (mass matrix). Material non linear problems in static and dynamic situations.

Some more aspects about the finite element method

Flow-chart of a simple program for finite element analysis. Substructuring. Post-processing of the results. Accuracy of the solutions, reduced integration, hourglass modes, incompressible materials.

Practical activities

During the course, practical and theoretical exercises will be held with the aid of programs running on PCs to get the students confident with numerical techniques applied to the analysis of structures. Convergence tests and critical results assessment.

Assessment method

Development of a project related to FE implementation or performing of FE analyses, followed by and oral examination.

Propaedeuticity

Analisi A-B, Analisi C, Geometria, Meccanica Razionale, Scienza delle Costruzioni (Structural Mechanics).

References/Course material

1. R. Brighenti. “Analisi numerica dei solidi e delle strutture: introduzione al metodo degli elementi finiti”, Esculapio Editore, III Ed., 2019.
2. Cook, R.D., Malkus D.S., Plesha, M.E.: “Concept and application of finite element analysis”, 4^{th} edition, John Wiley & Sons, 2002.
3. Zienkiewicz, O.C.: “The finite element method”, Mc Graw-Hill, 2000.
4. Corradi dell’Acqua, L.: "Meccanica delle strutture", Vol. 1,2 e 3, Mc Graw-Hill, 1995.

Other references

1. Hughes, T.J.R.: “The finite element method. linear static and dynamic finite element analysis”, Prentice Hall, 1987.
2. Owen, D.R.J., Hinton, E.: “Finite elements in plasticity”, Pineridge Press, 1980.
3. Bathe, K.J., “Finite element procedures”, Prentice Hall, 1996.