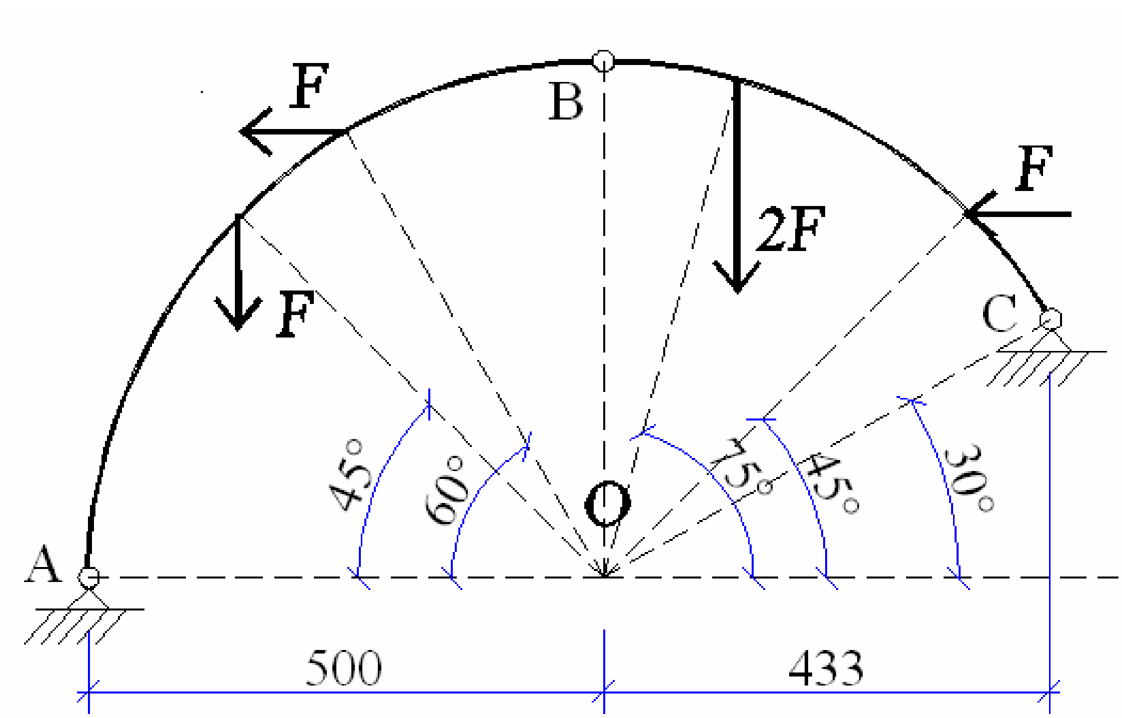
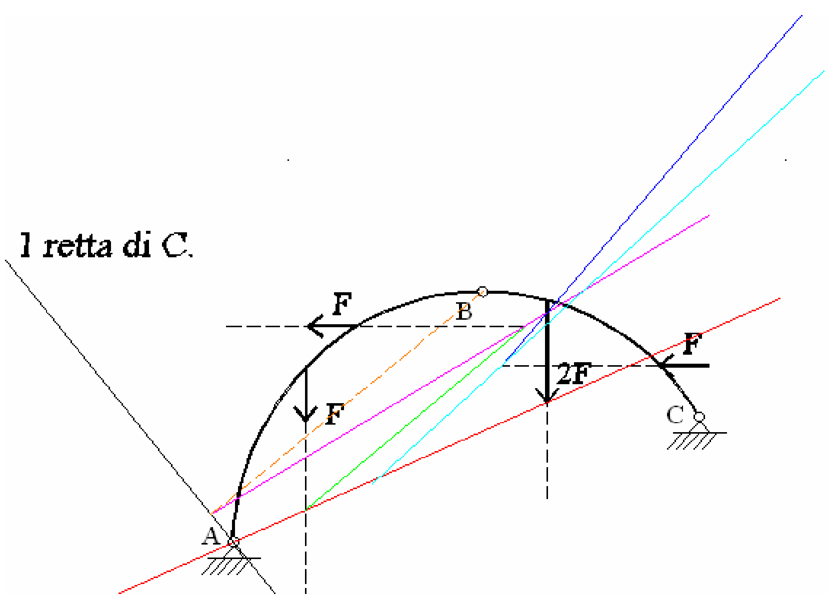
SCIENZA DELLE COSTRUZIONI II: TEORIA DELLE STRUTTURE TRASPARENTI

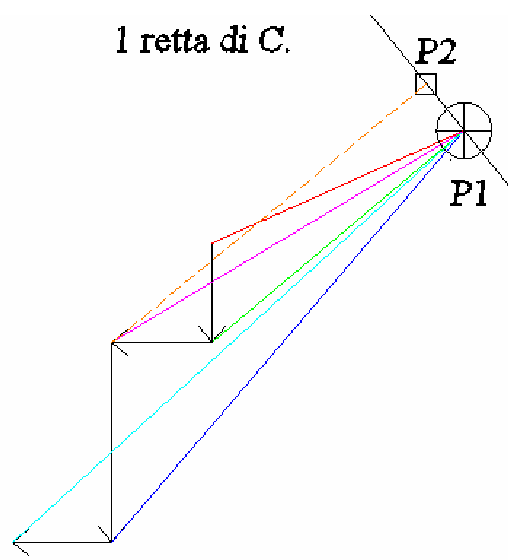
Prof. Brighenti - Prova scritta del **12/02/2015**

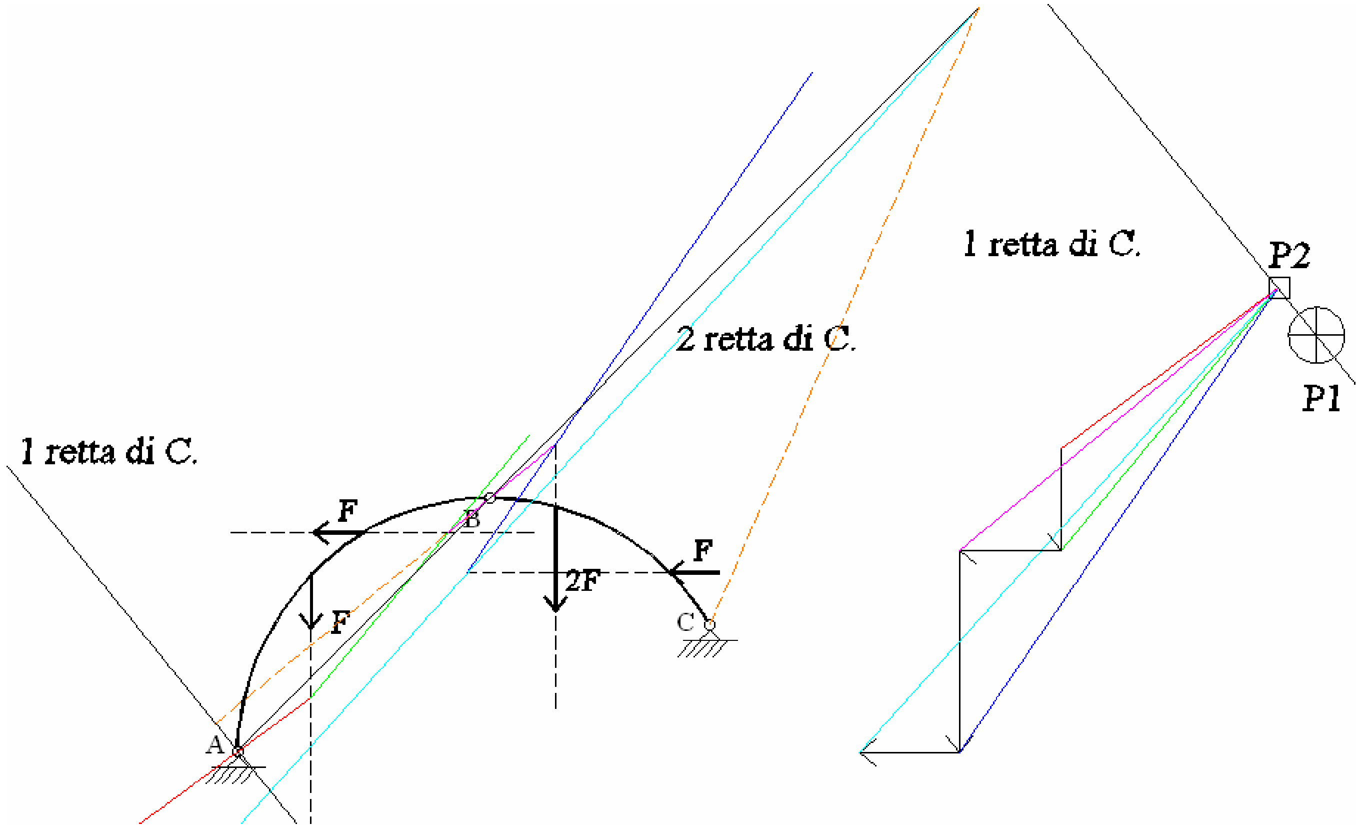
SOLUZIONE

1)

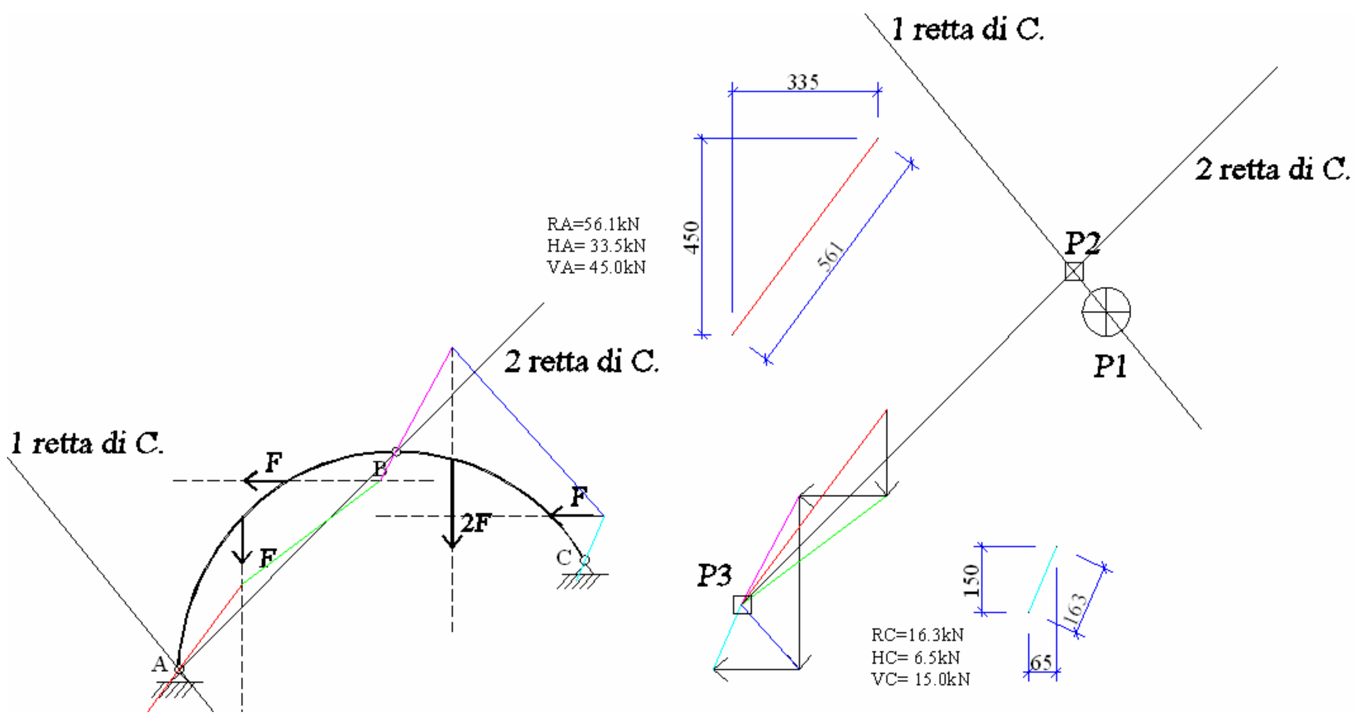






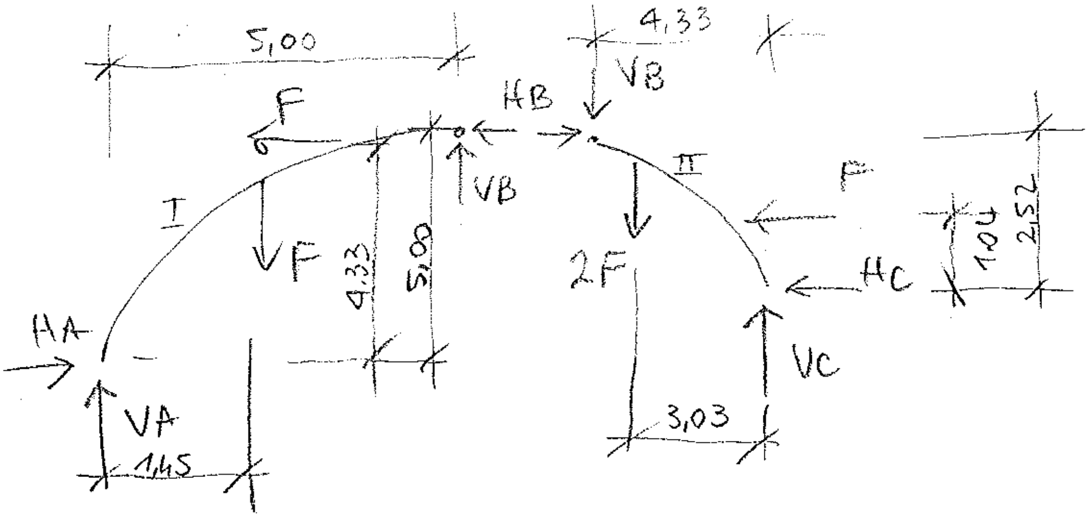


1 retta di C.
2 retta di C.
1 retta di C.
P2
P1
F
B
F
2F
F
C
A



1 retta di C.
335
2 retta di C.
450
561
RA=56.1kN
HA= 33.5kN
VA= 45.0kN
P2
P1
2 retta di C.
1 retta di C.
F
B
F
2F
F
C
A
P3
150
163
65
RC=16.3kN
HC= 6.5kN
VC= 15.0kN

$F = 20\,kN$



$$\text{I}\begin{cases} HA - F - HB = 0 \\ VA - F + VB = 0 \\ \curvearrowright_A\; F\cdot 1{,}45 - F\cdot 4{,}33 - VB\cdot 5{,}0 - HB\cdot 5{,}0 = 0 \end{cases}$$

$\underline{VA} = F - VB = 20 + 24{,}97 \simeq \underline{45\ kN}$

$\rightarrow HB = \dfrac{-57{,}6}{5{,}0} - VB = -11{,}52 - VB$

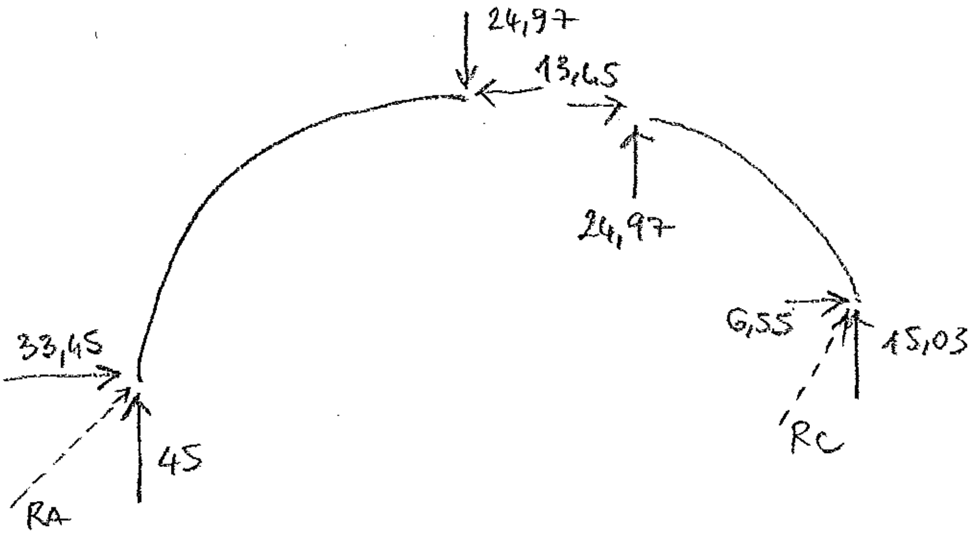
$$\text{II}\begin{cases} HB - F - HC = 0 \\ -VB - 2F + VC = 0 \\ \curvearrowright_C :\ HB\cdot 2{,}52 - VB\cdot 4{,}33 - 2F\cdot 3{,}03 - F\cdot 1{,}04 = 0 \end{cases}$$

$-29{,}03 - 2{,}52\,VB - 4{,}33\cdot VB - 142 = 0$

$\underline{VB} = -\dfrac{171{,}03}{6{,}85} = \underline{-24{,}97}\ kN \qquad \underline{HB} = -11{,}52 + 24{,}97 = \underline{13{,}45\ kN}$

$\underline{HC} = HB - F = \underline{-6{,}55\ kN}$

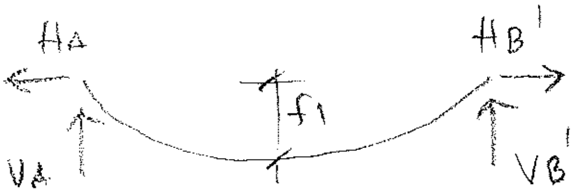
$\underline{HA} = F + HB = 20 + 13{,}45 = \underline{33{,}45\ kN} \qquad \underline{VC} = 2F + VB = 40 - 24{,}97 = \underline{15{,}03\ kN}$

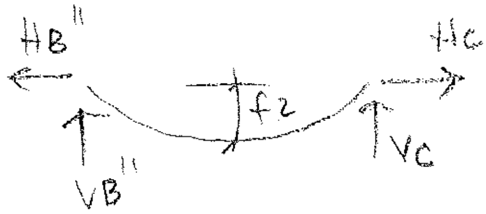


RA= 56 KN

RC= 16,4 KN

2)





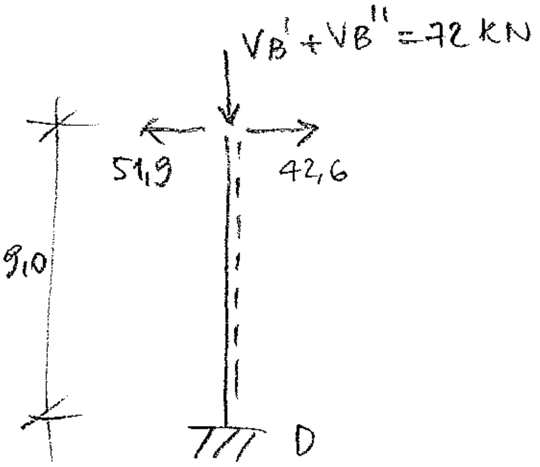
$$f_1 = \frac{\sqrt{l_1}}{4}\sqrt{6(L_1-l_1)} = 1{,}56\ m \qquad f_2 = \frac{\sqrt{l_2}}{4}\sqrt{6(L_2-l_2)} = 1{,}27\ m$$

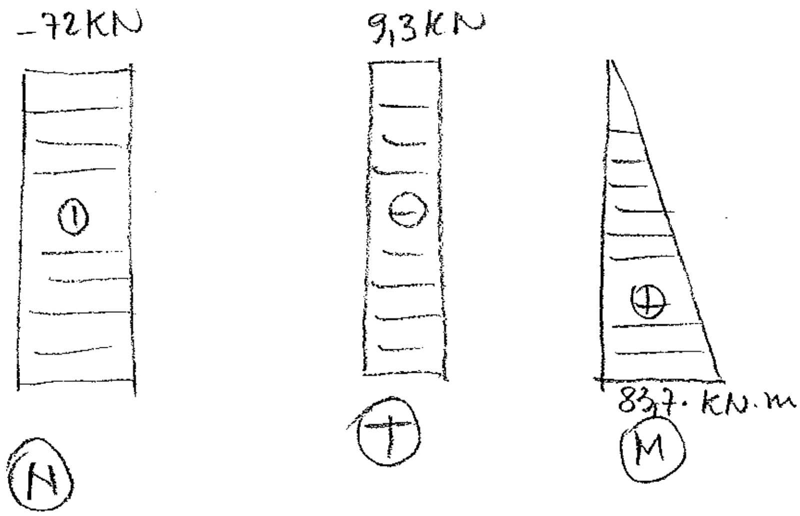
$$H_A = H_B' = \frac{q_1\, l_1^2}{8 f_1} = \frac{8\cdot 9^2}{8\cdot 1{,}56} = 51{,}9\ KN \qquad H_C = H_B'' = \frac{q_2\cdot l_2^2}{8 f_2} = 42{,}6\ KN$$

$$V_A = q_1\frac{l_1}{2} = 36\ KN = V_B' \qquad V_C = V_B'' = q_2\frac{l_2}{2} = 36\ kN$$

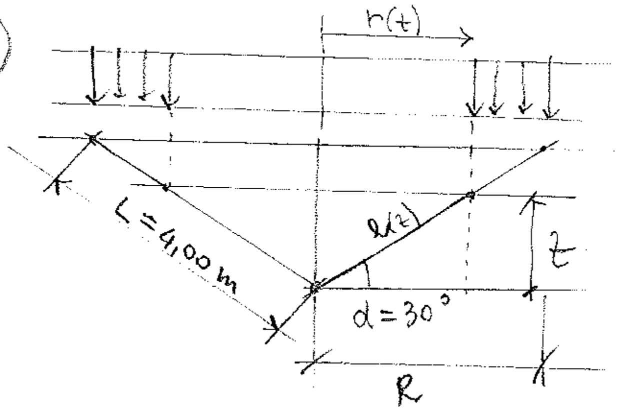
$$T_{max}^1 = \sqrt{H_A^2 + V_A^2} = 63{,}16\ KN \qquad T_{max}^2 = \sqrt{H_C^2 + V_C^2} = 55{,}77\ KN$$

$$\sigma_1 = \frac{T_{max}^1}{A\phi 26} = 1{,}18\cdot 10^8\ Pa \qquad \sigma_2 = \frac{T_{max}^2}{A\phi 26} = 1{,}05\cdot 10^8\ Pa$$





3)



$$\ell(z) = \frac{z}{\sin\alpha} \qquad r(z) = \frac{z}{tg\,\alpha}$$

$$\ell(z) = 2 \cdot z \qquad r(z) = 1{,}73 \cdot z$$

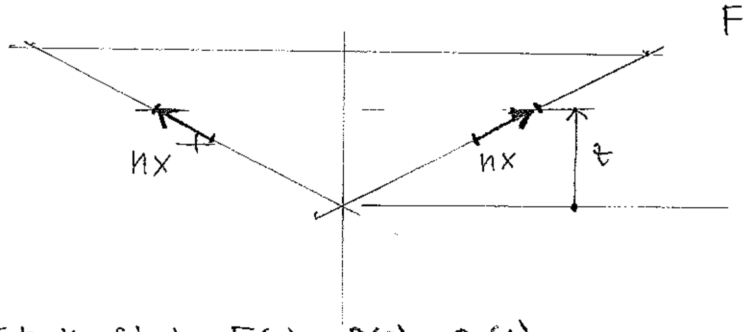
peso porzione di copertura $\left(S_L(z) = S_0 - S(z) = \pi L R - \pi \ell(z) \cdot r(z) = 43{,}48 - 10{,}87 \cdot z^2 \ m^2 \right)$

$$P(z) = \gamma \cdot 0{,}1 \cdot S_L(z)$$

$$= 24000 \cdot 0{,}1 \cdot (43{,}48 - 10{,}87\, z^2) \ N$$

$$Q(z) = q \cdot (\pi R^2 - \pi r^2) = \pi q (11{,}97 - 3 z^2) = 8000 \, (11{,}97 - 3 z^2) \ N$$

$$F(z) = P(z) + Q(z)$$



$$2\pi r \cdot n_x \sin\alpha = F(t) = P(z) + Q(t)$$

$$2\pi \frac{z}{tg\,\alpha} n_x \cdot \sin\alpha = \left[2400 \, (43{,}48 - 10{,}87 z^2) + 8000 \, (11{,}97 - 3 z^2) \right]$$

per $z = 1\,m \rightarrow \sigma_x = \frac{n_x}{t} = \frac{150024 \cdot tg\,\alpha}{2\pi \cdot 1 \cdot \sin\alpha} \frac{1}{t} = \frac{27571}{t} = 2{,}75 \cdot 10^5 \ Pa$ tensione di meridiano

$\sigma_x \to \infty$ per $z = 0$ (*)

$$\cancel{\frac{n_x}{R_x}} + \frac{n_y}{R_y} = -p_z \qquad R_x = \infty \qquad R_y = \frac{r}{\sin\alpha} \qquad p_z = (\gamma t + q) \cos\alpha$$

$$= (2400 + 8000) \cdot 0{,}87 = 9006 \ N/m^2$$

$$n_y = R_y \cdot (\gamma t + q) \cos\alpha$$

$$n_y = \frac{r(t)}{\sin\alpha} \cdot 9006 \cdot \cos\alpha \qquad \text{per } z = 1m: \ n_y = \frac{1{,}73 \cdot 9006 \cdot 0{,}87}{0{,}5} = 26986 \ N$$

$\sigma_y = \frac{n_y}{t} = 2{,}70 \cdot 10^5 \ Pa$ tensione di parallelo (*)

max per $z = 2{,}0\,m \rightarrow \sigma_y = 5{,}40 \cdot 10^5 \ Pa$

$\sigma_{max\ comp} = 2 \cdot 10^6 \ Pa \qquad \sigma_{max\ traz} = 1 \cdot 10^5 \ Pa$

(*) la verifica non si può eseguire essendo $\sigma_x \to \infty$ $\sigma_y \to \infty$: NO VERIFICA per $z = 0$