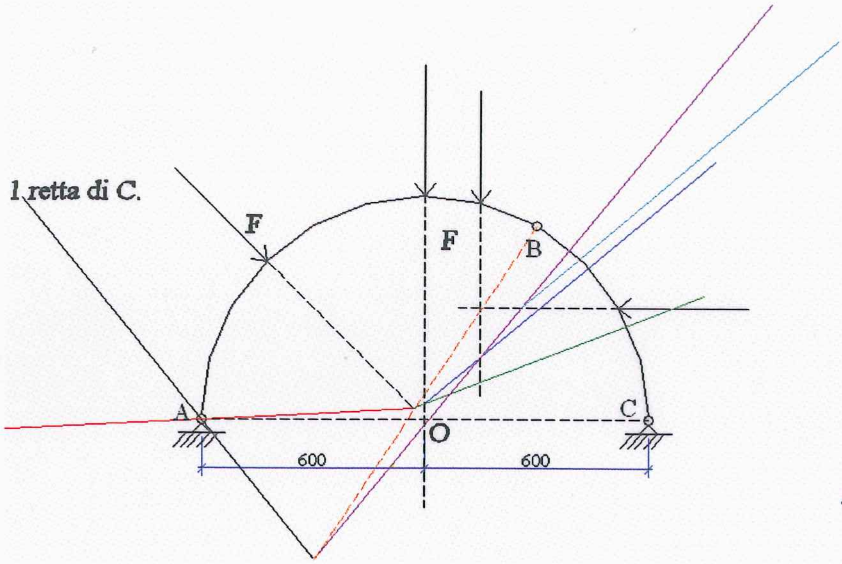
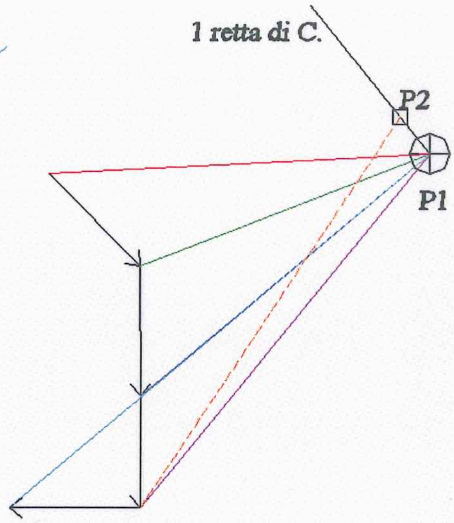
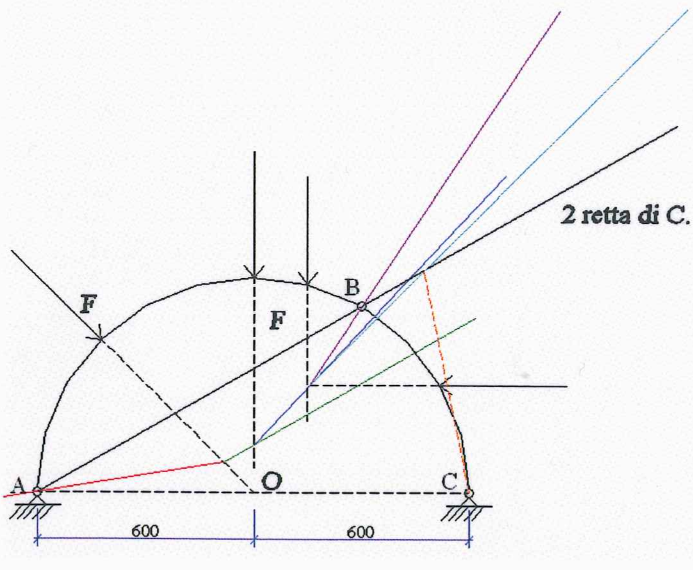
SCIENZA DELLE COSTRUZIONI II: TEORIA DELLE STRUTTURE TRASPARENTI

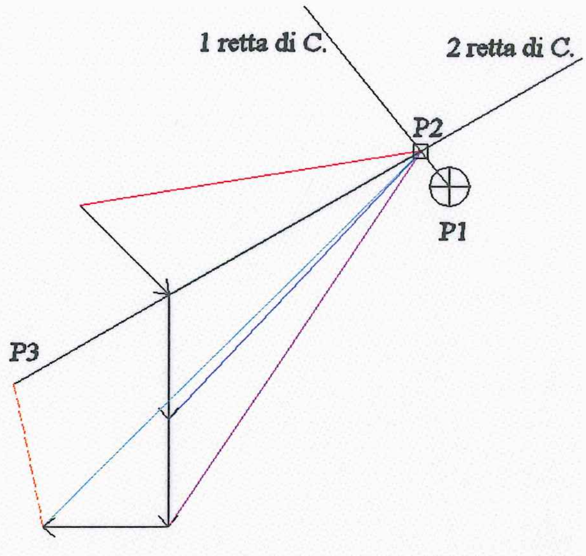
Soluzione della Prova scritta del 04/12/2014

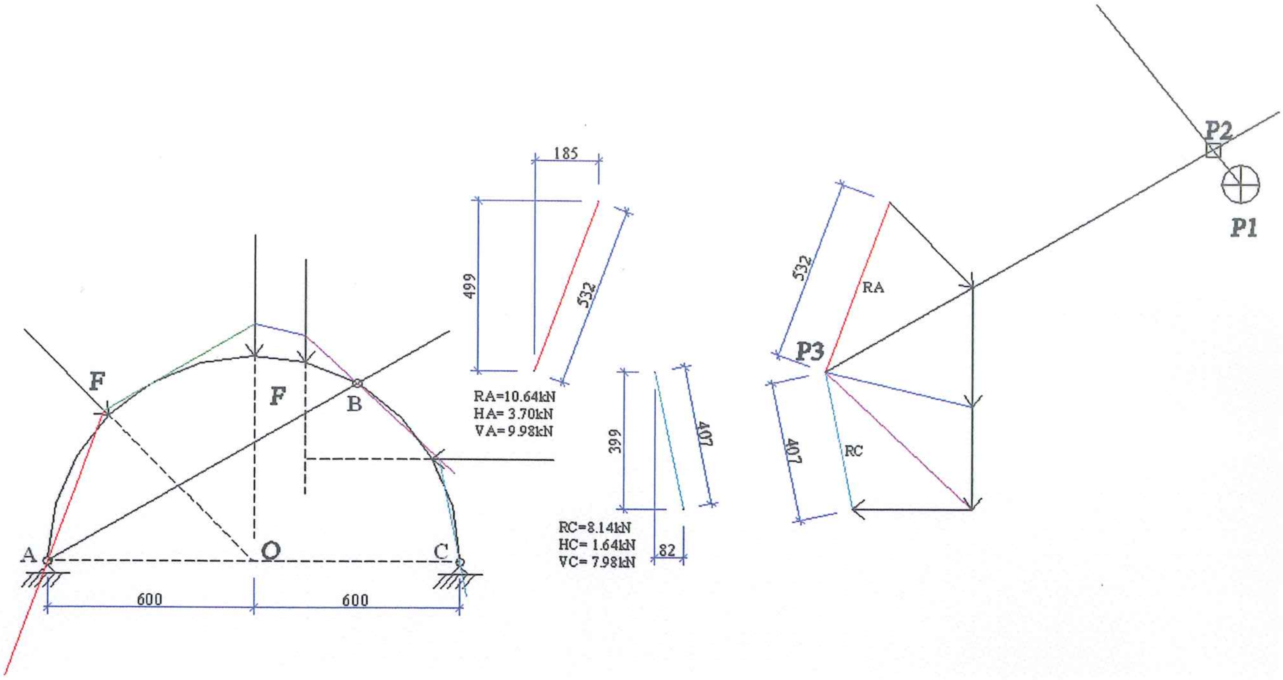
1. Esercizio



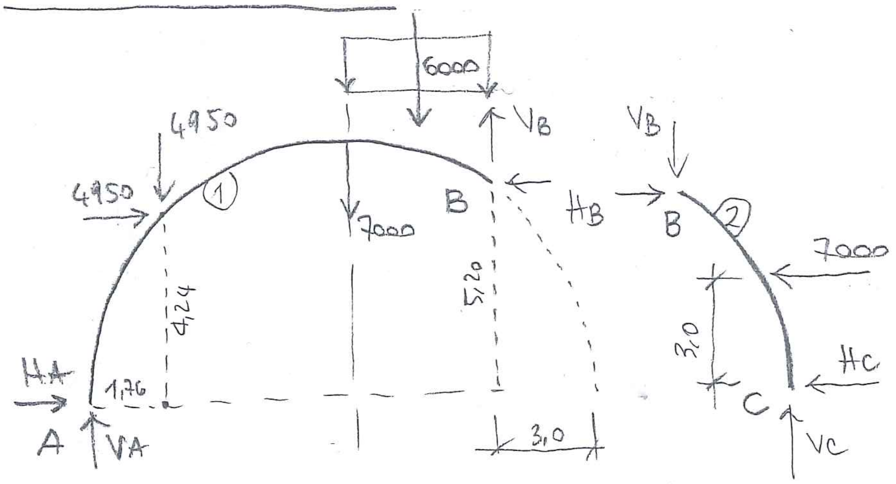








Soluzione analitica



Risoluzione con equazioni cardinali della statica

$$1)\begin{cases} HA + 4950 - HB = 0 \\ VA - 4950 - 7000 - 6000 + VB = 0 \\ \overset{\curvearrowleft}{A}\; 4950 \cdot 4{,}24 + 4950 \cdot 1{,}76 + 7000 \cdot 6 + 6000 \cdot 7{,}5 - VB \cdot 9 - HB \cdot 5{,}20 = 0 \end{cases}$$

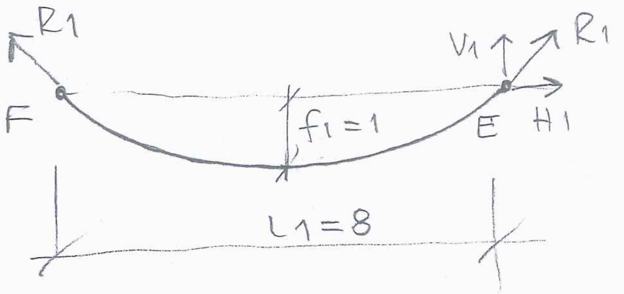
$$2)\begin{cases} HB - 7000 - Hc = 0 \\ -VB + Vc = 0 \\ \overset{\curvearrowleft}{C}\; HB \cdot 5{,}20 - VB \cdot 3 - 7000 \cdot 3 = 0 \end{cases}$$

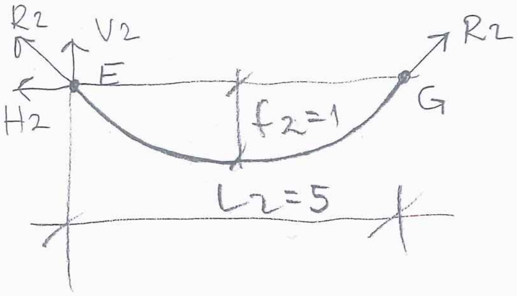
$HB = 8639\,N \qquad VB = 7974\,N$

$HA = HB - 4950 = 3689\,N \qquad VA = 9975\,N$

$Hc = HB - 7000 = 1639\,N \qquad Vc = VB = 7974\,N$

2)





$q_1 = q_2 = 5000$ N/m

$$V_1 = \frac{q_1 \cdot L_1}{2} = 20'000\,N \qquad V_2 = \frac{q_2 \cdot L_2}{2} = 12500\,N$$

$$H_1 = \frac{q_1 L_1^2}{8 f_1} = 40'000\,N \qquad H_2 = \frac{q_2 L_2^2}{8 f_2} = 15625\,N$$

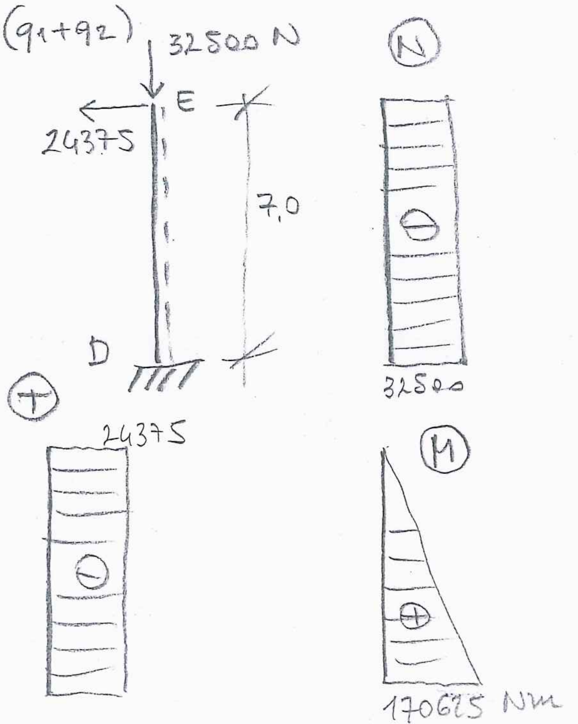
$$R_1 = \sqrt{H_1^2 + V_1^2} = 44721\,N \qquad R_2 = \sqrt{H_2^2 + V_2^2} = 20'009\,N$$

$$\sigma_1 = \frac{44721}{\pi \cdot \frac{20^2}{4}} = 142\,MPa \qquad \sigma_2 = \frac{20'009}{\pi \cdot \frac{20^2}{4}} = 63{,}7\,MPa$$

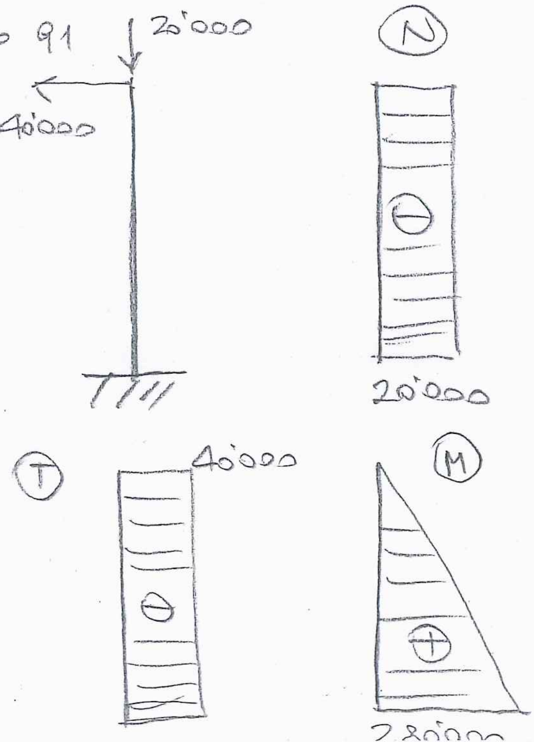
$$L_{sv} = L_1 + \frac{q_1^2 L_1^3}{24 H_1^2} = 8 + 0{,}33 = 8{,}33\,m \qquad L_{sv} = L_2 + \frac{q_1^2 L_2^3}{24 H_2^2} = 5 + 0{,}53 = 5{,}53\,m$$

<u>Studio pilastro DE</u>

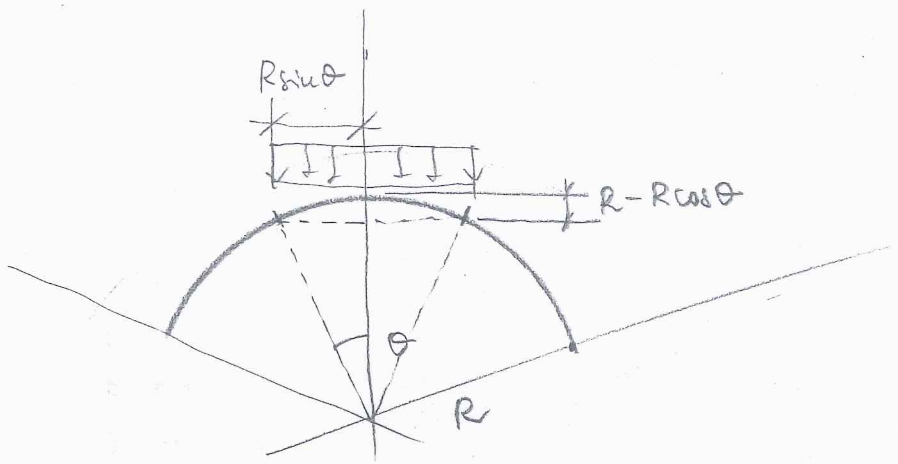
$(q_1 + q_2)$



solo q_1

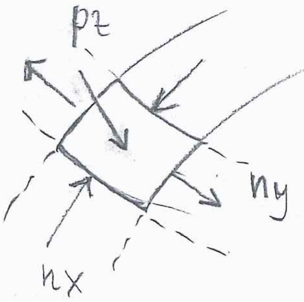


3)



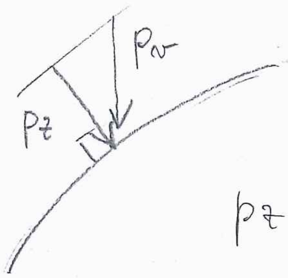
peso calotta : $P(\theta) = 2\pi R^2(1-\cos\theta)\, s \cdot \gamma = 2 \cdot \pi R^2 (1-\cos\theta)\, 0{,}12 \cdot 19000 =$
$= 14325 \cdot R^2 (1-\cos\theta)$

Sforzi membranali ; $\quad n_x = -\dfrac{P(\theta)}{2\pi R \sin^2\theta} = -\dfrac{\gamma s R}{1+\cos\theta}$



$$n_x = -\frac{P(\theta)}{2\pi R \sin^2\theta} = \frac{-14325 \cdot R^{\cancel{2}}(1-\cos\theta)}{2\pi \cancel{R} \sin^2\theta}$$

$$\frac{n_x}{R_x} + \frac{n_y}{R_y} = -p_z \rightarrow n_y = -n_x - p_z \cdot R \quad (*)$$



$$p_z = p_v \cdot \cos\theta = \gamma \cdot s \cdot \cos\theta$$

$$(*) \quad n_y = \frac{\gamma s R}{1+\cos\theta} - \gamma s \cos\theta\, R$$

$\sigma_x = \dfrac{|n_x|}{s} \leq 5 \cdot 10^5$ Pa ; $\quad \sigma_y = \dfrac{n_y}{s} \leq 8 \cdot 10^4$ Pa $\quad$ scelgo la più restrittiva, R_{min}

$$\sigma_x = \frac{|n_x|}{s} = \frac{\gamma s R}{(1+\cos\theta) \cdot s} = \frac{19000 \cdot s \cdot R}{(1+\cos 60^\circ) \cdot s} = \frac{1520\, R}{s} \leq 5 \cdot 10^5 \rightarrow R \leq 39{,}47 \text{ m}$$

$$\sigma_y = \left|\frac{n_y}{s}\right| \leq 8 \cdot 10^4 \rightarrow \left[\frac{\gamma s R}{1+\cos\theta} - \gamma s R \cos\theta\right]\frac{1}{s} = \left[\frac{19000 \cdot R}{1{,}5} - 19000\, R \cdot 0{,}5\right] \leq 8 \cdot 10^4 \rightarrow \underline{R \leq 25 \text{ m}}$$