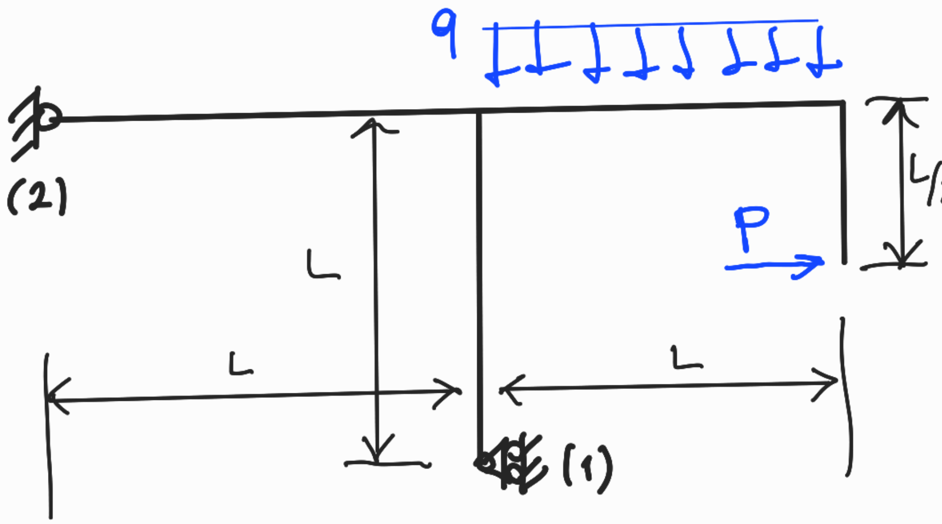
Soluz. Compito Fondam. di Scienza delle Costruz. 18.07.2023

1) • Verifica che la struttura è isostatica

n aste = 1 $\quad v = 2+1 = 3$

$g = 3$ → isostatica

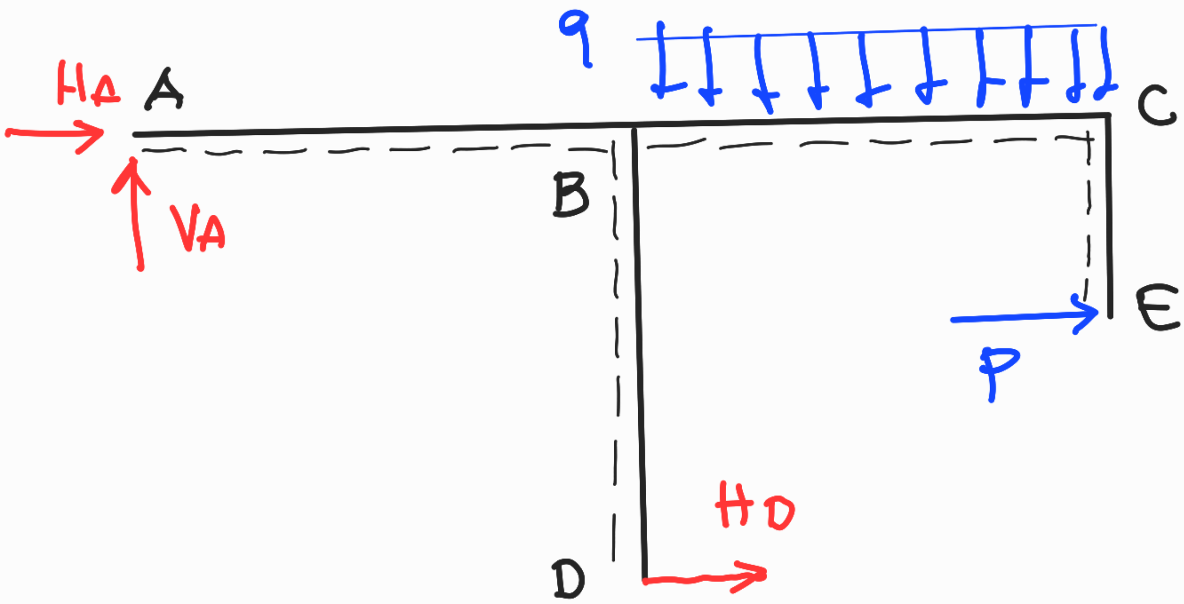


$L = 5$ m

$q = 9\,000$ N/m

$P = 7'000$ N

• Calcolo delle reazioni vincolari



Equilibri delle aste

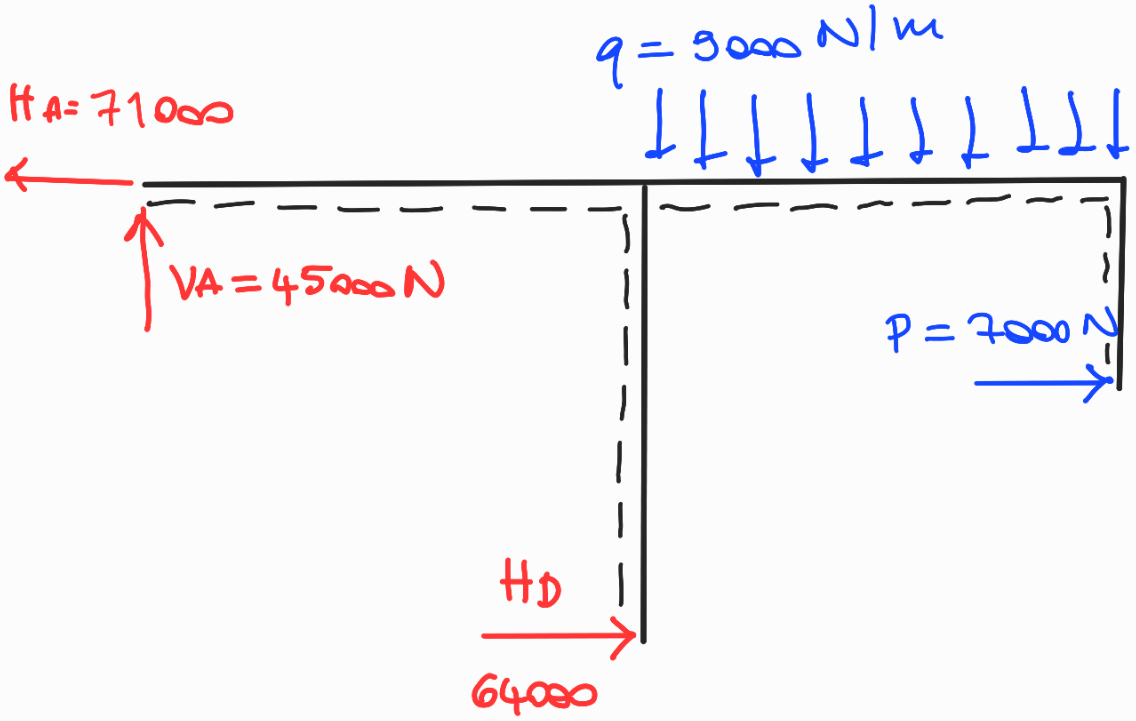
$$\begin{cases} \overset{+}{\rightarrow} & H_A + H_D + P = 0 \\ \uparrow & V_A - qL = 0 \\ \curvearrowright_A & qL\,\frac{3}{2}L - H_D \cdot L - P\frac{L}{2} = 0 \end{cases}$$

$$V_A = qL = 45000\ N$$

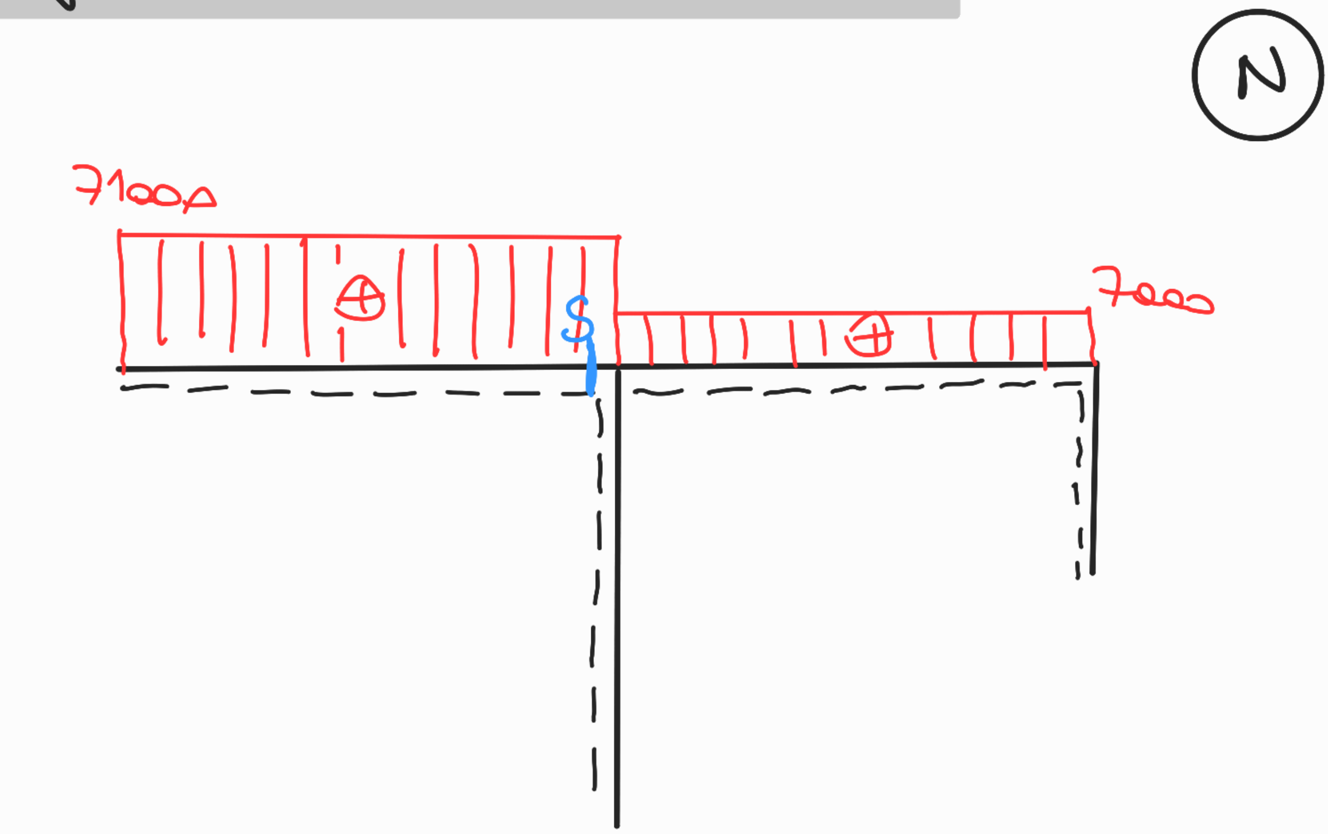
$$H_D = \frac{3}{2}qL - \frac{P}{2} = 64000\ N$$

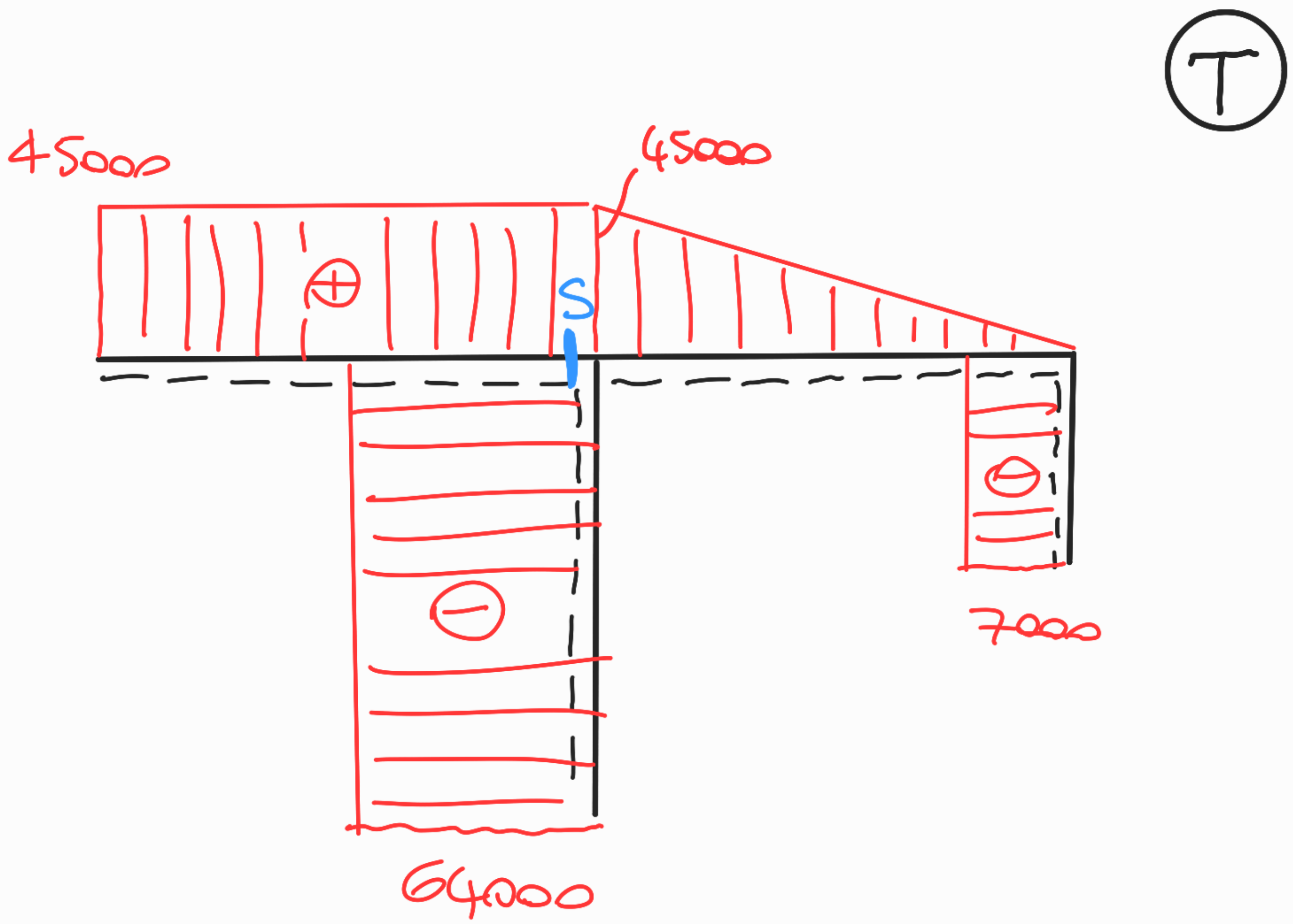
$$H_A = -P - H_D = -71000\ N$$

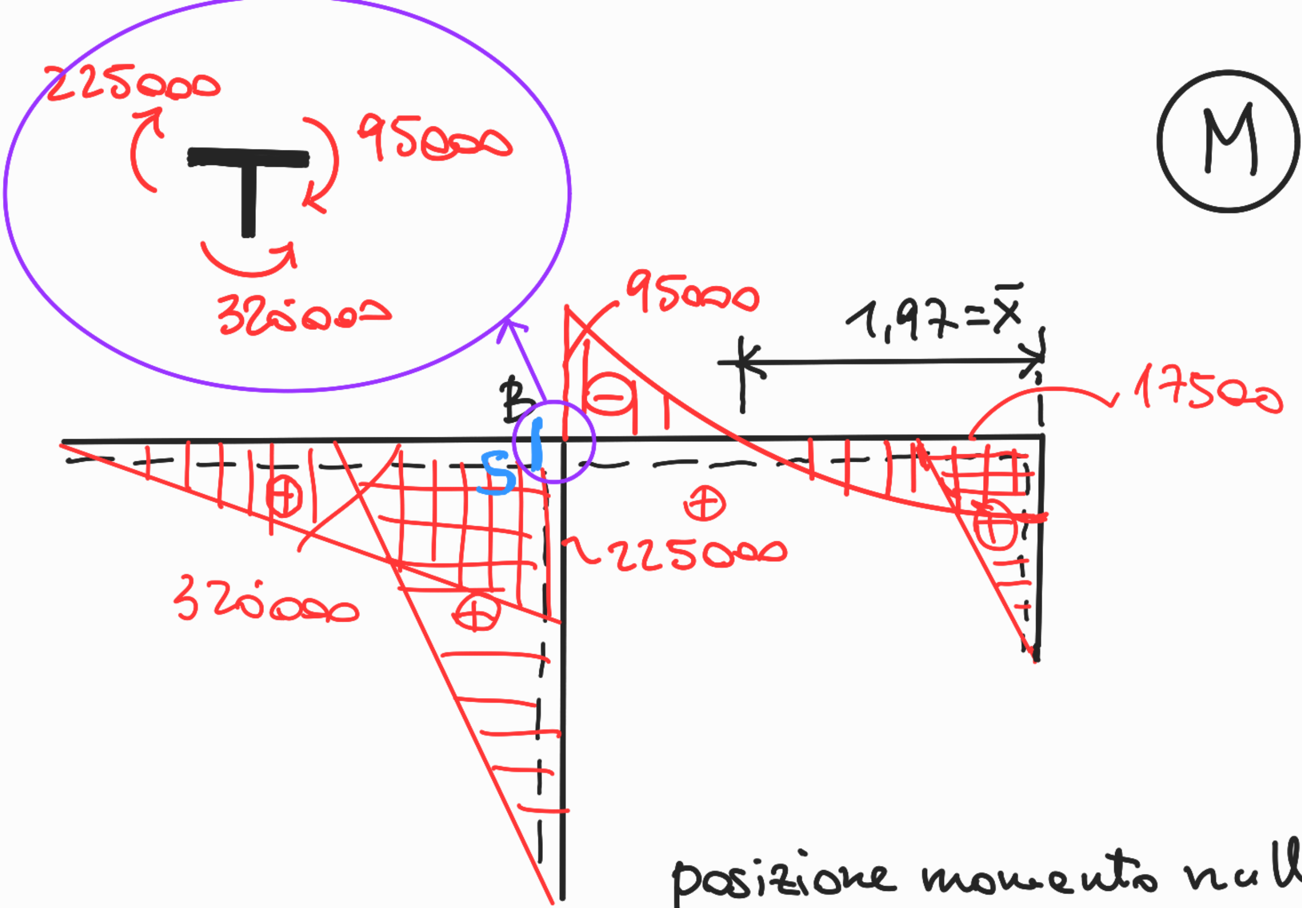
Schema finale delle forze attive e reattive



- **Diagrammi delle azioni interne**





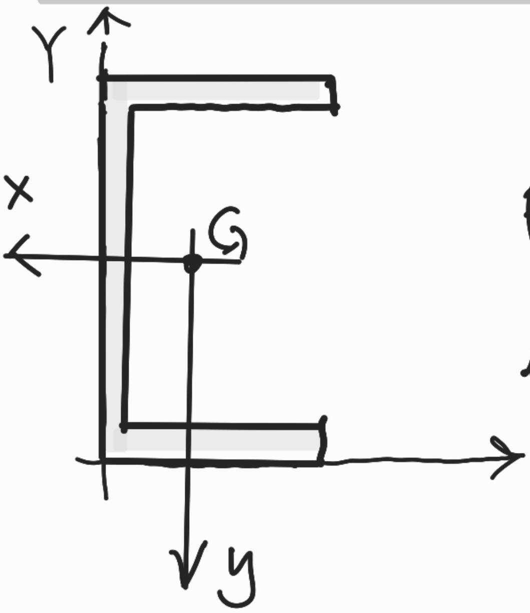


posizione momento nullo:

$$q\frac{\bar{x}^2}{2} - P\cdot\frac{L}{2} = 0 \rightarrow \bar{x} = 1,97$$

2)

- Caratteristiche geometriche della sezione



$$A = 2(0,15\cdot 0,01) + 0,3\cdot 0,01 = 0,006\,m^2$$

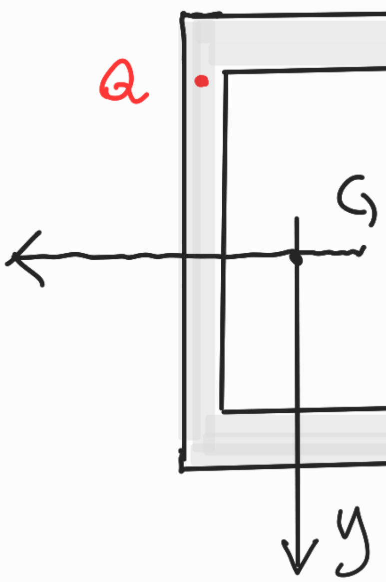
Posizione baricentro

$$\begin{cases} Y_G = 0,15 \\ X_G = \dfrac{S_Y}{A} = \dfrac{0,15\cdot 0,01\cdot 0,075\cdot 2}{0,006} = \\ \quad = 0,0375\,m \end{cases}$$

$$I_x = \frac{0,01\cdot 0,3^3}{12} + \left(\frac{0,15\cdot 0,01^3}{12} + 0,15\cdot 0,01\cdot 0,075^2\right)\cdot 2 \cong 9\cdot 10^{-5}\,m^4$$

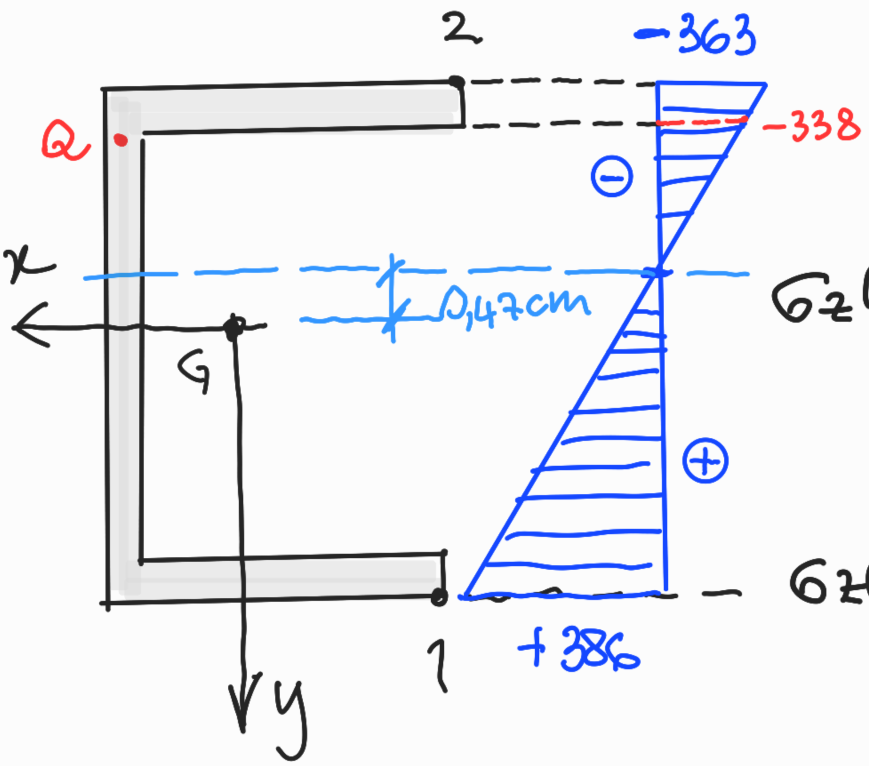
Il momento di inerzia I_y non è necessario.

- Calcolo delle Tensioni nelle sez. S-S



$$\begin{cases} N = +71000 \ N \\ T = 45000 \ N \\ M_x = 225\,000 \ Nm \end{cases}$$

- Pressoflessione



$$\sigma_z(y) = \frac{N}{A} + \frac{M_x}{I_x} y$$

$$\sigma_z(1) = \frac{71000}{6 \cdot 10^{-3}} + \frac{225000}{9 \cdot 10^{-5}} \cdot 0{,}15 =$$

$$= 386 \ MPa$$

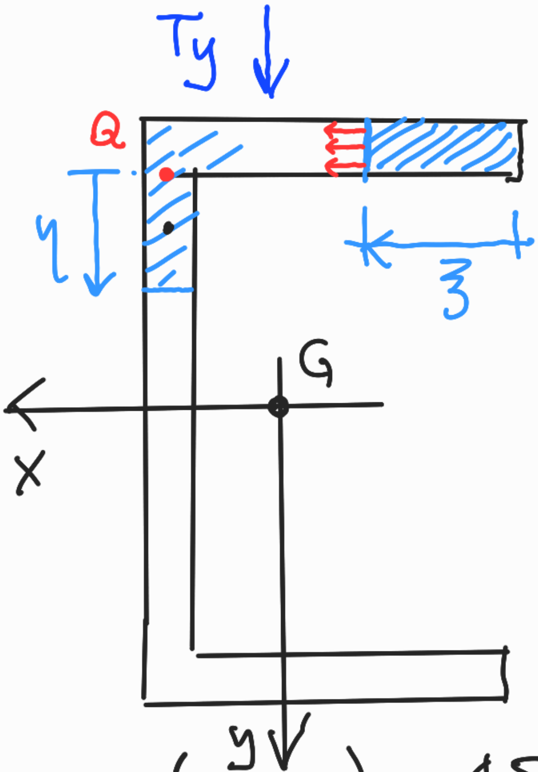
$$\sigma_z(2) = \frac{71000}{6 \cdot 10^{-3}} + \frac{225000}{9 \cdot 10^{-5}} (-0{,}15) =$$

$$= -363 \ MPa$$

$$\sigma_z(Q) = \frac{71000}{6 \cdot 10^{-3}} + \frac{225000 \, (-0{,}14)}{9 \cdot 10^{-5}} = -338 \ MPa$$

• **Taglio** (f. di Jourawski)

$$\tau_{zs} = -\frac{T_y \, S_x^*}{I_x \cdot b}$$



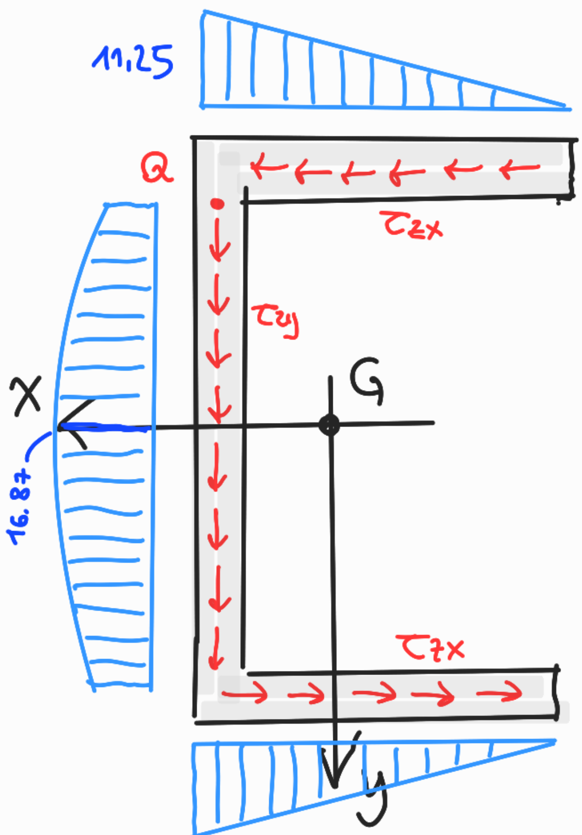
Si assume che il taglio T_y passi per il centro di taglio

$$\tau_{zx}(\zeta) = \frac{-45000 \cdot (0{,}01 \cdot \zeta \cdot (-0{,}15))}{9 \cdot 10^{-5} \cdot 0{,}01}$$

$$\tau_{zx}(\zeta = 0{,}15) = -\frac{45000\,(0{,}01 \cdot 0{,}15\,(-0{,}15))}{9 \cdot 10^{-5} \cdot 0{,}01} = 11{,}25 \text{ MPa}$$

$$\tau_{zx}(Q) = 11{,}25 \text{ MPa}$$

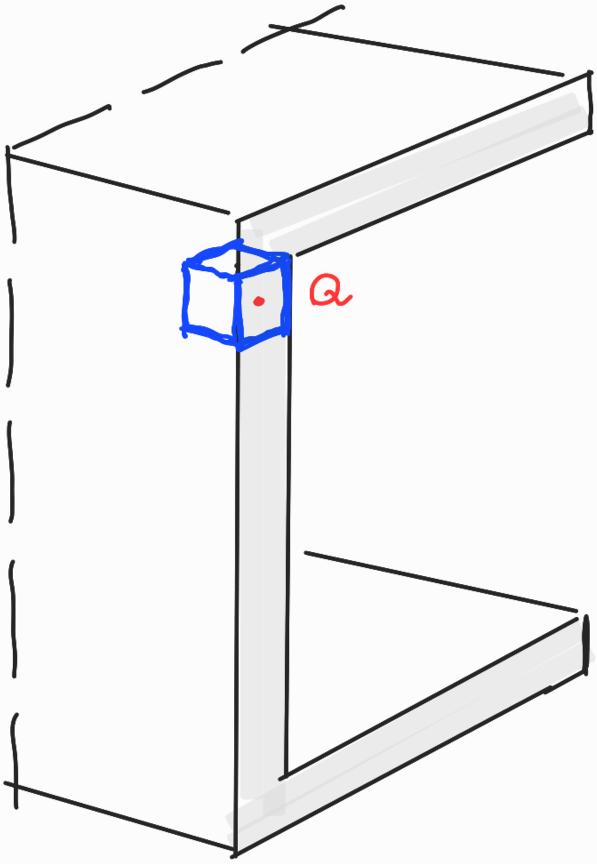
$$\tau_{zy}(\eta) = \frac{-45000 \cdot \left[-0{,}15 \cdot 0{,}01 \cdot 0{,}15 - 0{,}01 \cdot \eta \left(0{,}15 - \frac{\eta}{2}\right)\right]}{9 \cdot 10^{-5} \cdot 0{,}01}$$

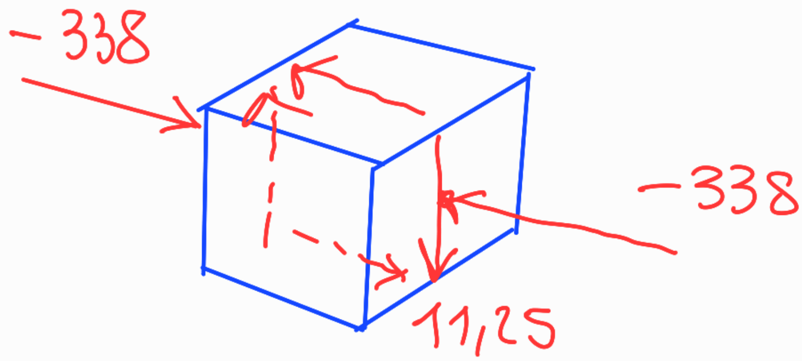


$$\tau_{zy}^{max} = \tau_{zy}(\eta = 0{,}15) = 16{,}87 \text{ MPa}$$

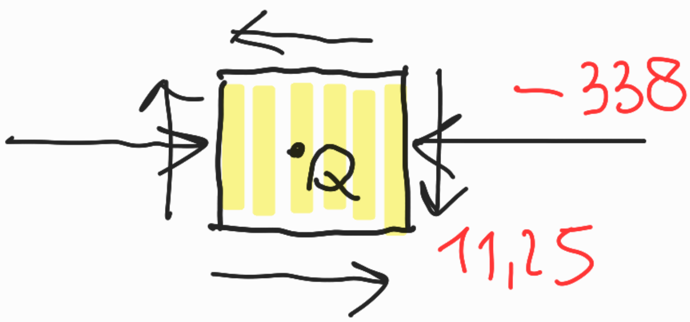
Distribuzione delle tensioni tangenziali nella sezione

- Verifica di resistenza nel punto Q della sez S e cerchio di Mohr





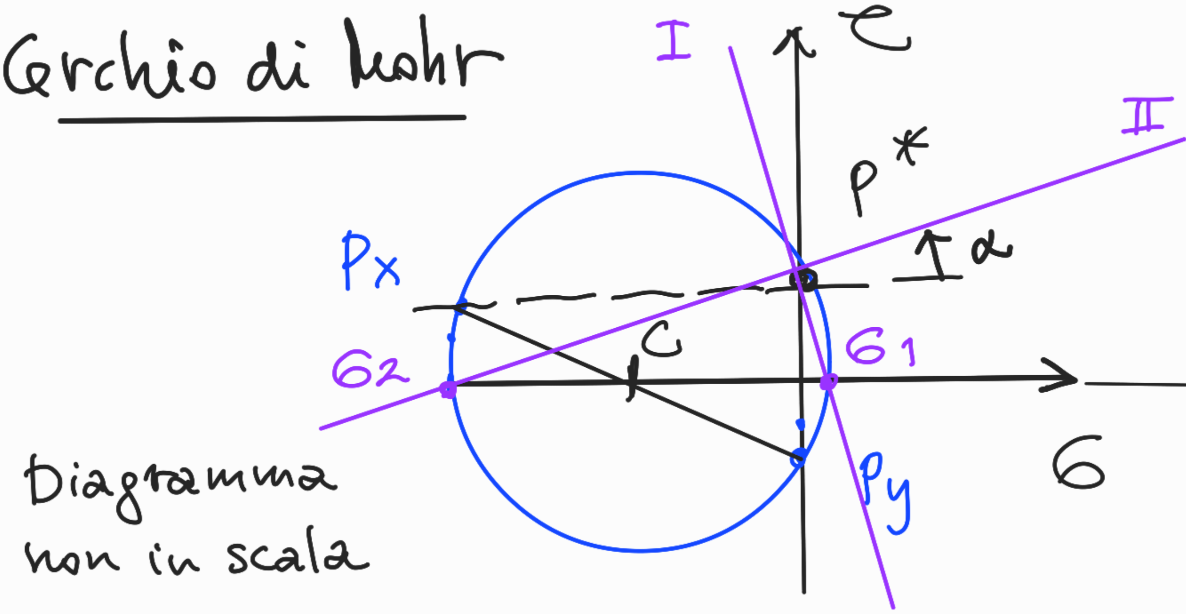
Punto Q (MPa) stato tensionale

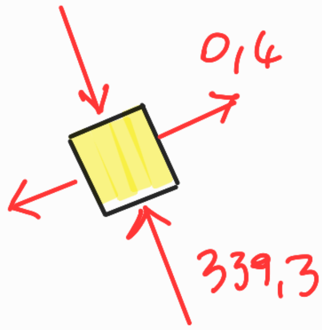


$P_x = (-338\,;\,11{,}25)$

$P_y = (0\,;\,-11{,}25)$

Cerchio di Mohr





$$x_c = \frac{-338+0}{2} = -169,5 \text{ MPa}$$

$$r = 169,8 \text{ MPa}$$

$$\begin{cases} \sigma_1 = x_c + r = +0,4 \text{ MPa} \\ \sigma_2 = x_c - r = -339,3 \text{ MPa} \end{cases}$$ Tensioni principali

$$\alpha = \frac{1}{2} \operatorname{arctg} \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = +3,8° \text{ antiorario}$$

- Verifica di resistenza nel punto Q

Criterio di Von Mises

$$\sigma_{eqVM} = \sqrt{\sigma^2 + 3\tau^2}$$

$$\sigma_{eqVM} = \sqrt{338^2 + 3 \cdot 11{,}25^2} = 338{,}6 \text{ MPa}$$

$$\sigma_{eqVM} = 338{,}6 \text{ MPa} < 400 \text{ MPa}$$

verificato