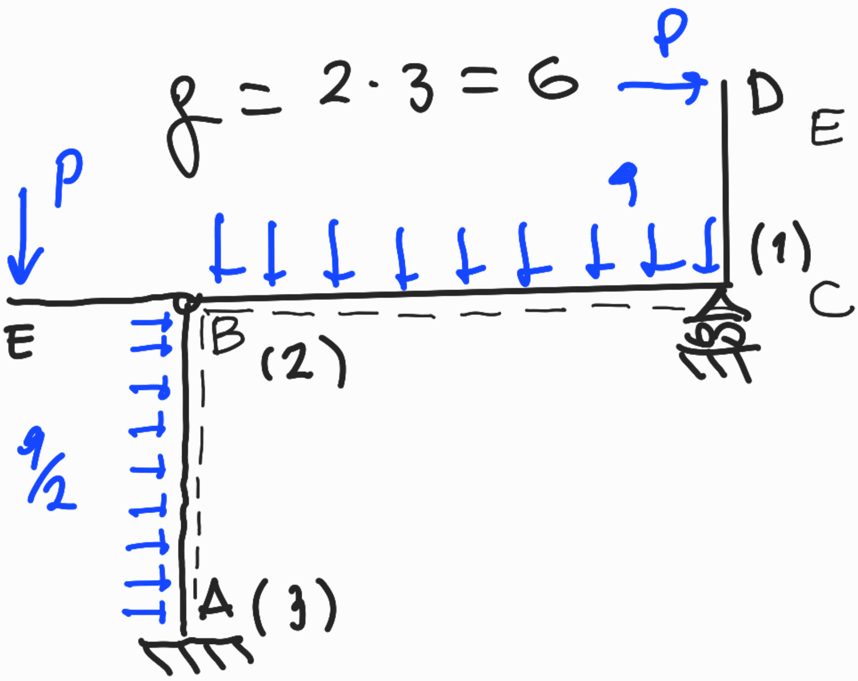
Soluz. Compito Fondam. di Scienza delle Costruz. 29.06.2023

1) • Verifica che la struttura è isostatica

$n_{aste} = 2$ $\qquad v = 3+2+1 = 6$

$\ell = 2 \cdot 3 = 6$ → isostatica

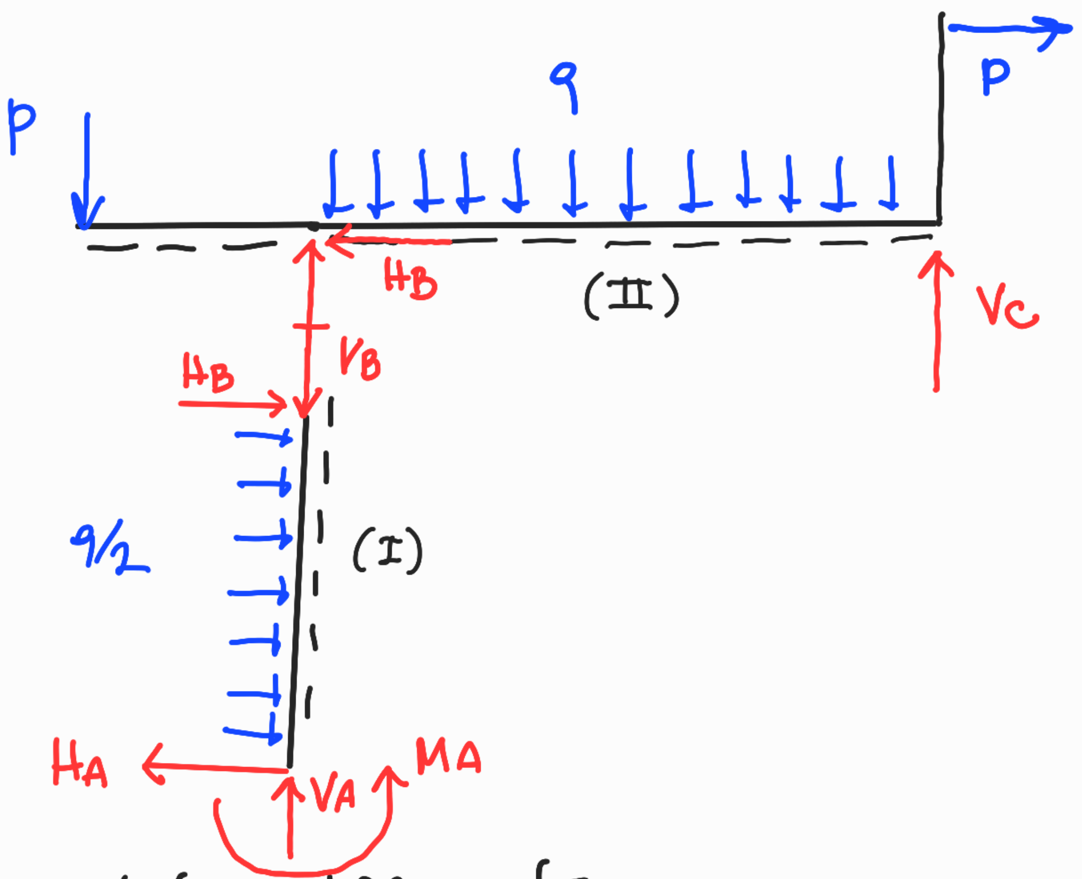


$L = 4$ m

$q = 8\,000$ N/m

$P = 10'000$ N

• Calcolo delle reazioni vincolari



Equilibri delle aste:

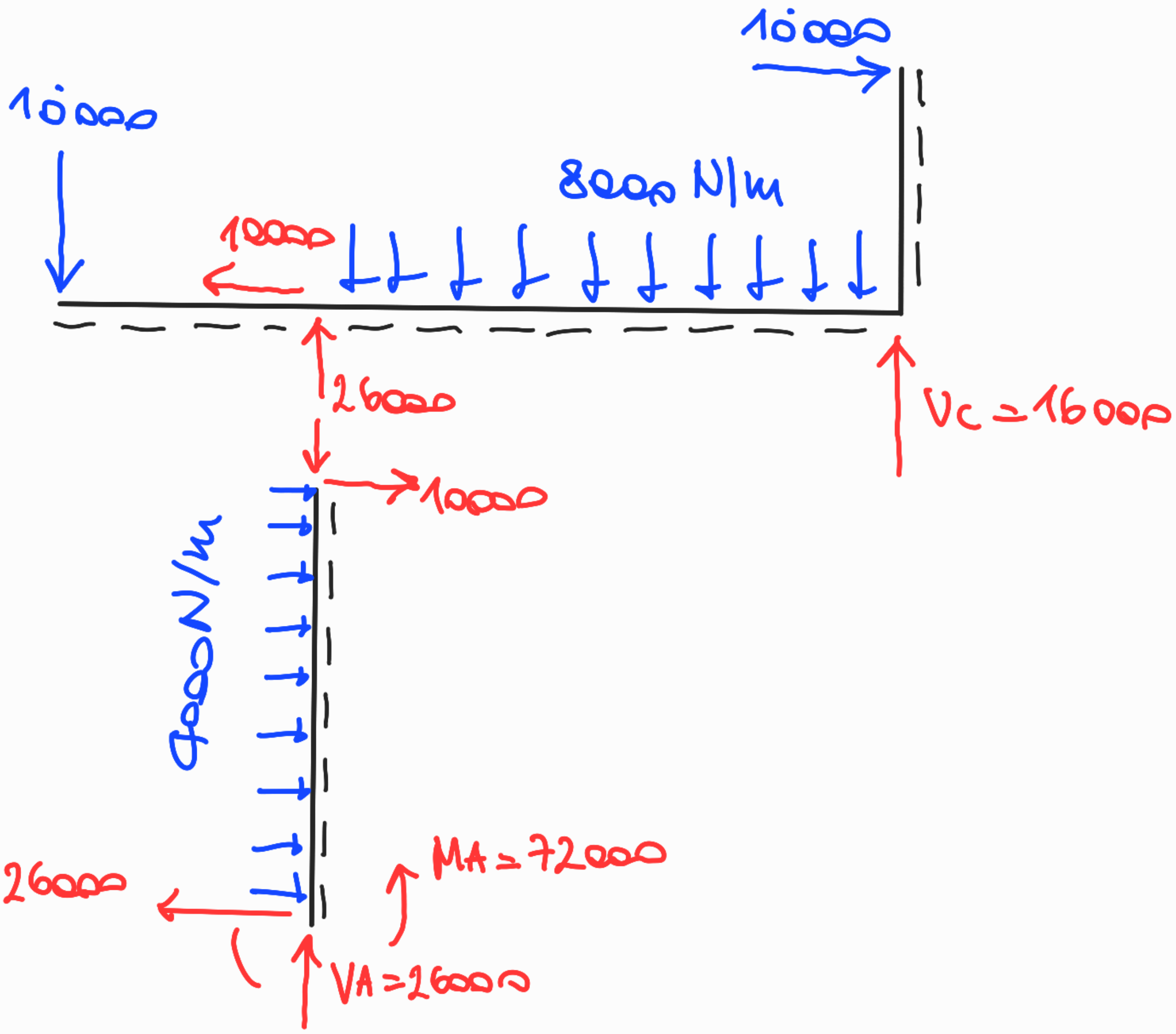
I

$$\begin{cases} \xrightarrow{+} \; -H_A + \frac{q}{2}\cdot L + H_B = 0 & H_A = H_B + \frac{q}{2} L = 26000 \text{ N} \\ \uparrow \; V_A - V_B = 0 & V_A = 26000 \text{ N} \\ \curvearrowright_A \; -M_A + \frac{q}{2}\frac{L^2}{2} + H_B\cdot L = 0 & M_A = \frac{qL^2}{4} + H_B\cdot L = 72000 \text{ Nm} \end{cases}$$

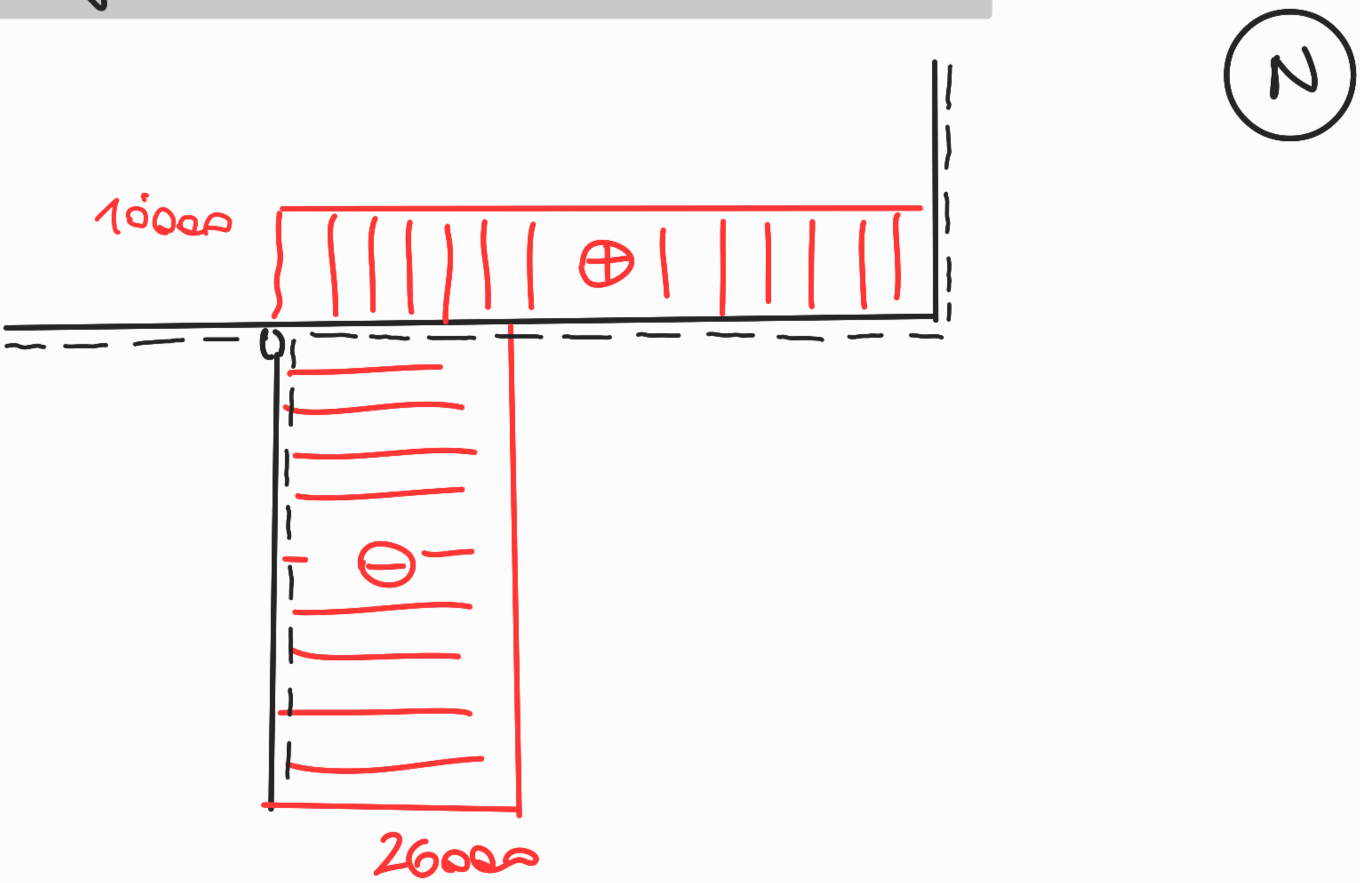
II

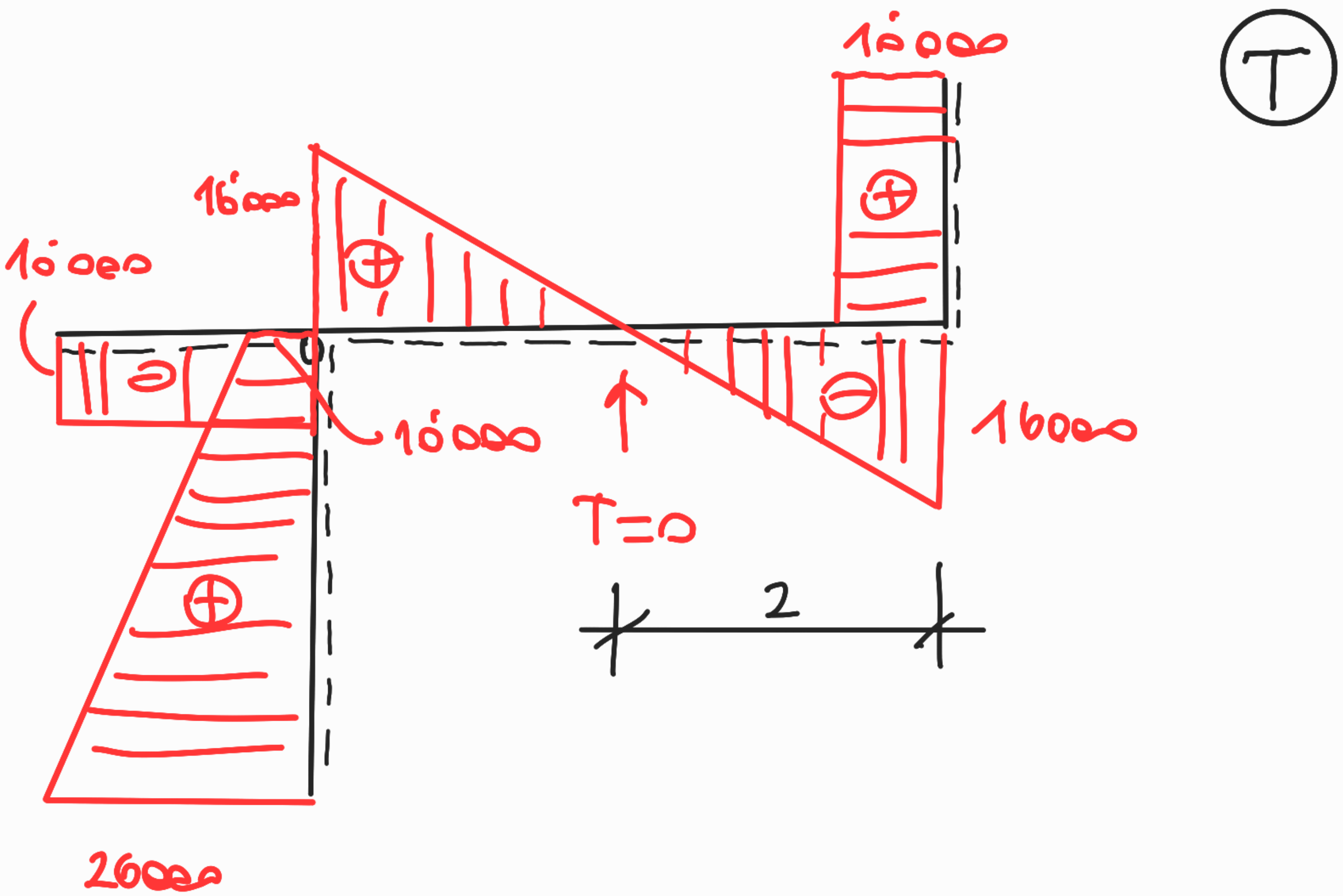
$$\begin{cases} \xrightarrow{+} \; -H_B + P = 0 & H_B = P = 10000 \text{ N} \\ \uparrow \; V_B - qL + V_C - P = 0 & V_B = P + qL - V_C = 26000 \text{ N} \\ \curvearrowright_B \; -P\frac{L}{2} + \frac{qL^2}{2} - V_C\cdot L + P\frac{L}{2} = 0 & V_C = +\frac{qL}{2} = 16000 \text{ N} \end{cases}$$

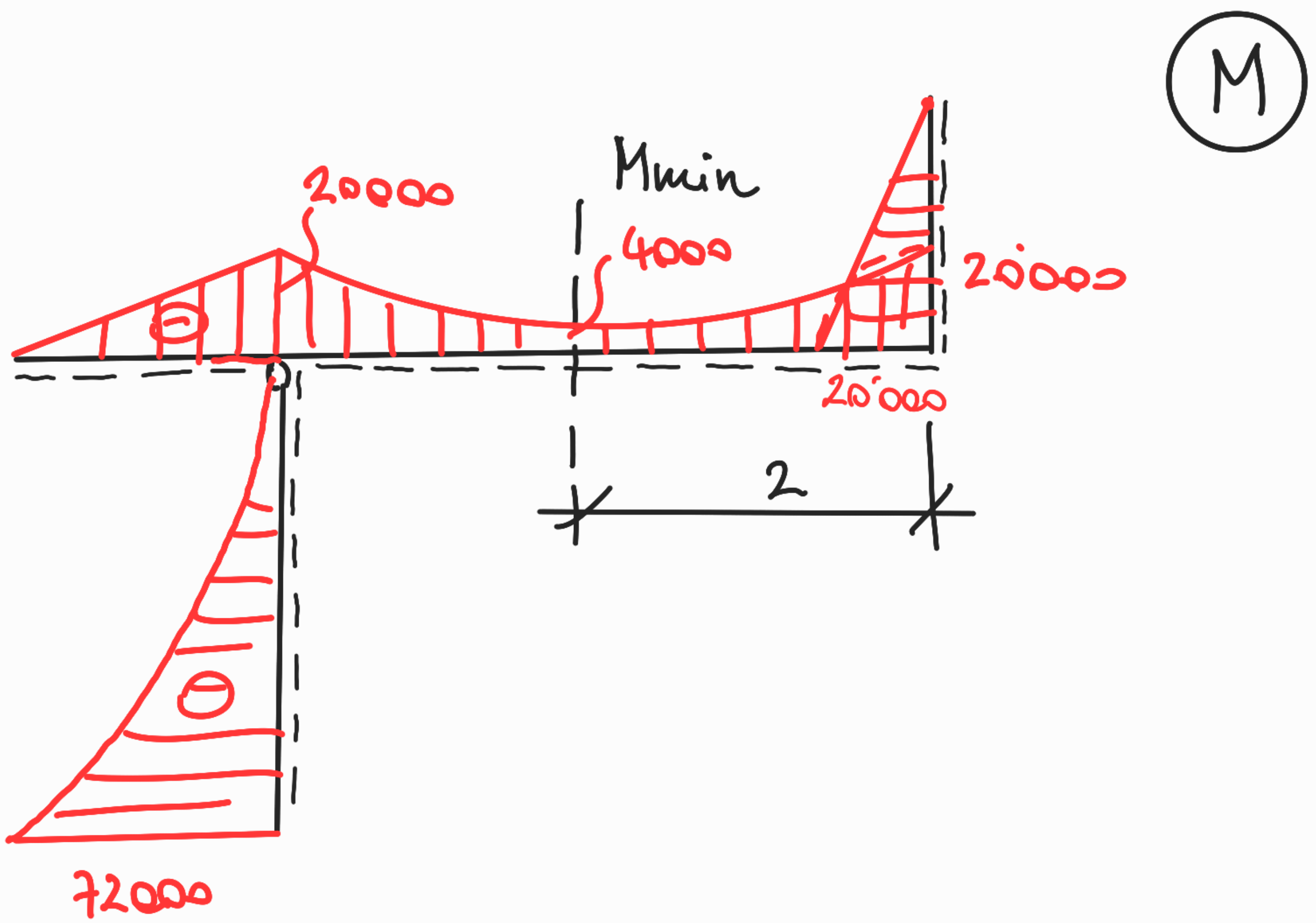
Schema finale delle forze attive e reattive



- Diagrammi delle azioni interne

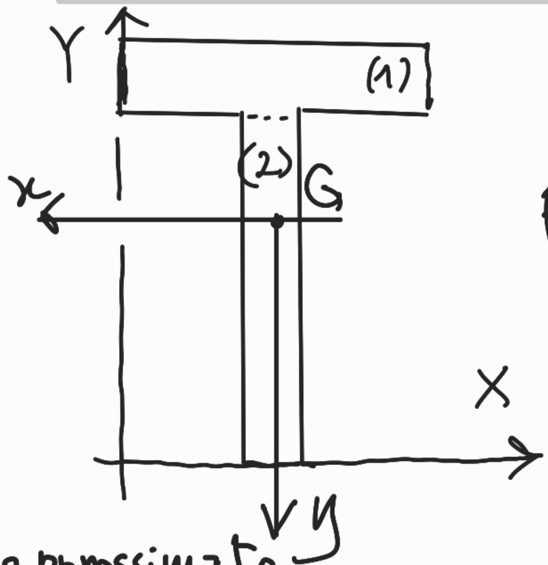






2)

- Caratteristiche geometriche della sezione



$A = 0{,}2 \cdot 0{,}02 + 0{,}4 \cdot 0{,}01 = 8 \cdot 10^{-3}\, m^2$

Posizione baricentro

$X_G = 0{,}1\, m$

$Y_G = \frac{S_x}{A} = \frac{0{,}2 \cdot 0{,}02 \cdot 0{,}4 + 0{,}4 \cdot 0{,}01 \cdot 0{,}2}{8 \cdot 10^{-3}}$

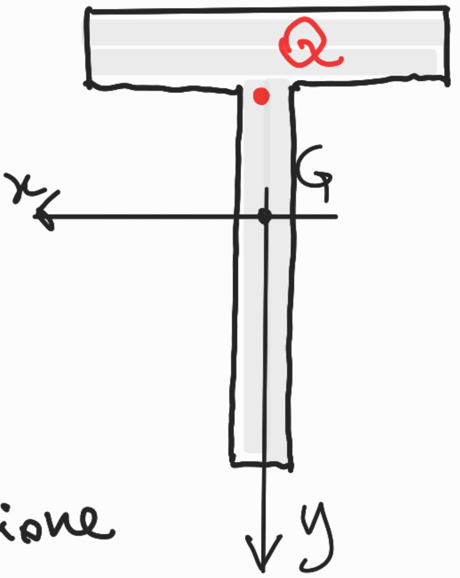
$= 0{,}3\, m$

approssimato ↓

$I_x = \frac{0{,}2 \cdot 0{,}02^3}{12} + 0{,}2 \cdot 0{,}02 \cdot 0{,}1^2 + \frac{0{,}01 \cdot 0{,}4^3}{12} + 0{,}4 \cdot 0{,}01 \cdot 0{,}1^2 = 1{,}33 \cdot 10^{-4}\, m^4$

Il momento di inerzia I_y non è necessario.

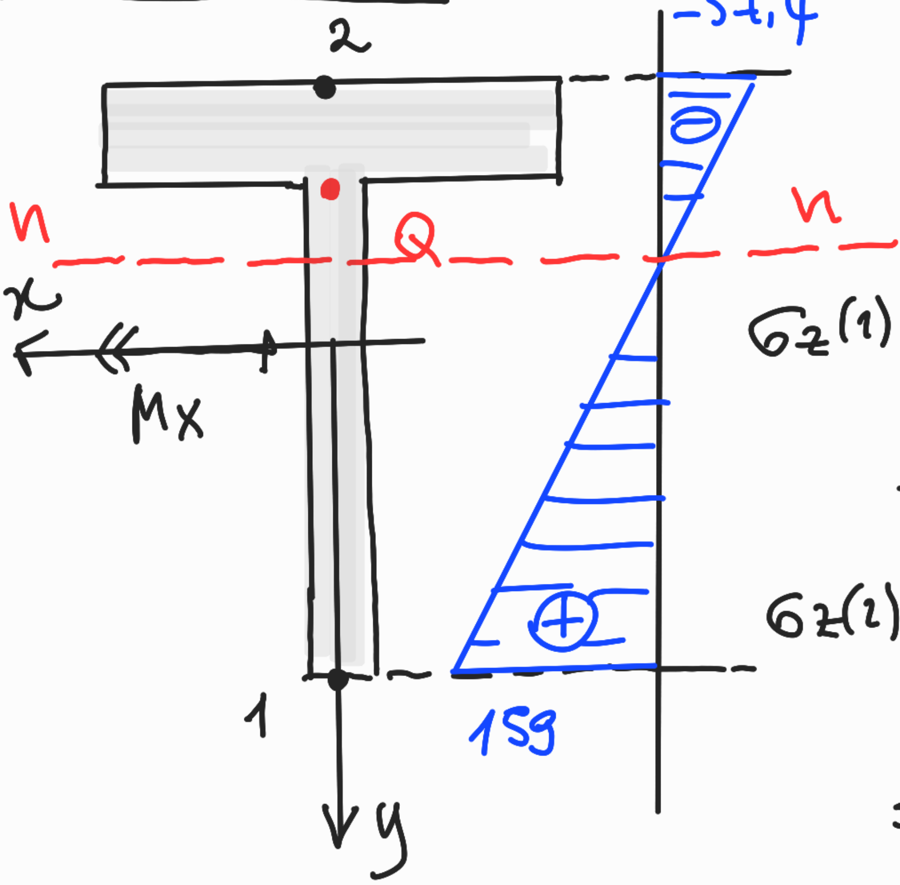
- Calcolo delle Tensioni nella sez. S-S



Sezione posta alla base del pilastro AB

$$\begin{cases} N = -26000 \ N \\ T = 26000 \ N \\ M_x = 72000 \ Nm \end{cases}$$

- Pressoflessione



$$\sigma_z(y) = \frac{N}{A} + \frac{M_x}{I_x} y$$

$$\sigma_z(1) = \frac{-26000}{8 \cdot 10^{-3}} + \frac{72000}{1{,}33 \cdot 10^{-4}} \cdot 0{,}3 =$$

$$= 159 \ MPa$$

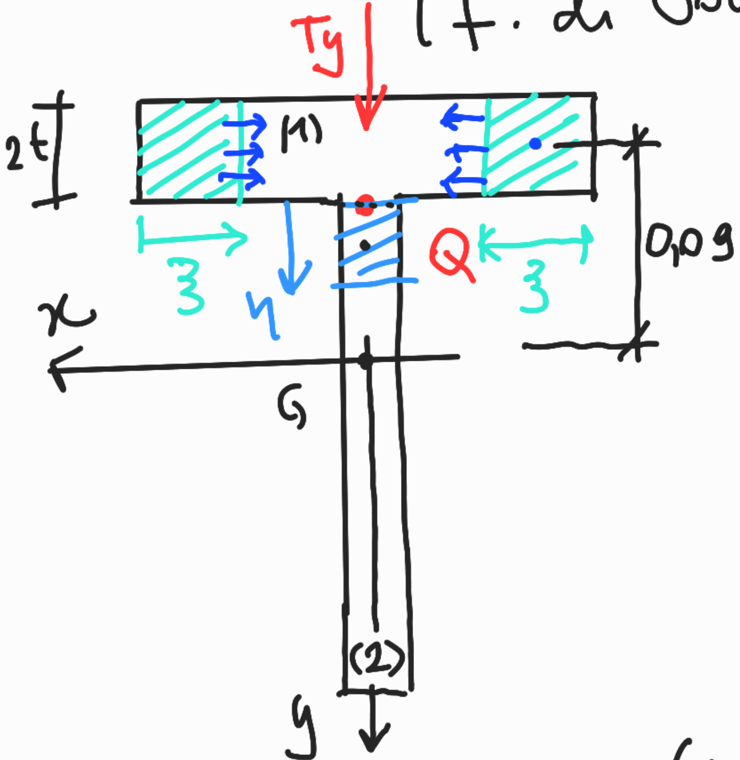
$$\sigma_z(2) = \frac{-26000}{8 \cdot 10^{-3}} + \frac{72000}{1{,}33 \cdot 10^{-4}} \cdot (-0{,}1) =$$

$$= -57{,}4 \ MPa$$

$$\sigma_z(Q) = -\frac{26000}{8 \cdot 10^{-3}} + \frac{72000}{1{,}33 \cdot 10^{-4}} (-0{,}09) = -51{,}9 \ MPa$$

- **Taglio**

(f. di Jourawski) $\tau_{zs} = -\dfrac{T_y \, S_x^*}{I_x \cdot b}$

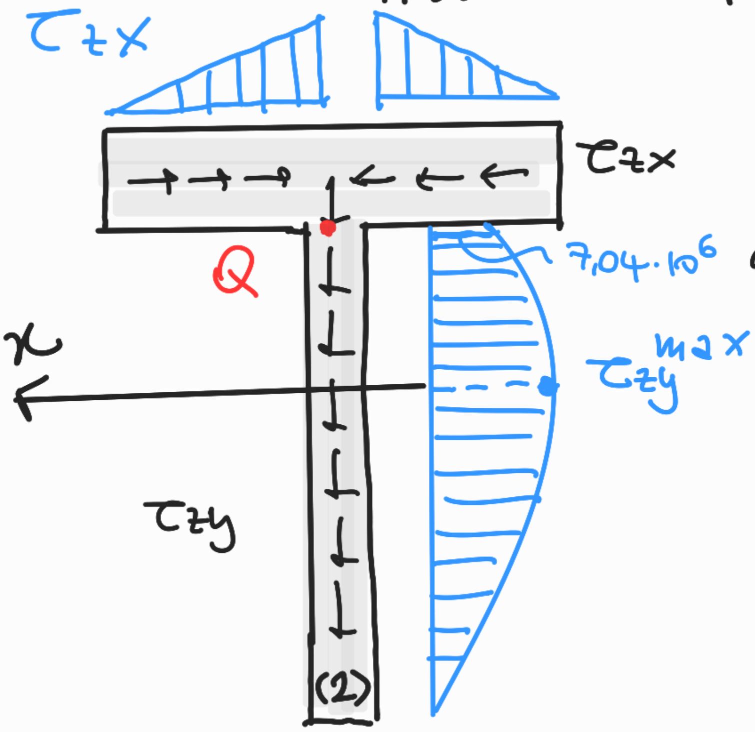


$$\tau_{zx}(\xi) = -\frac{T_y \cdot (2\,\xi \cdot 2t \cdot (-0{,}09))}{I_x \cdot 4t}$$

$$\tau_{zx}(\xi = 0{,}1) = -\frac{26000 \cdot (2 \cdot 0{,}1 \cdot 0{,}02 \cdot (-0{,}09))}{1{,}33 \cdot 10^{-4} \cdot 0{,}02 \cdot 2} = -7 \cdot 10^{4}\ Pa$$

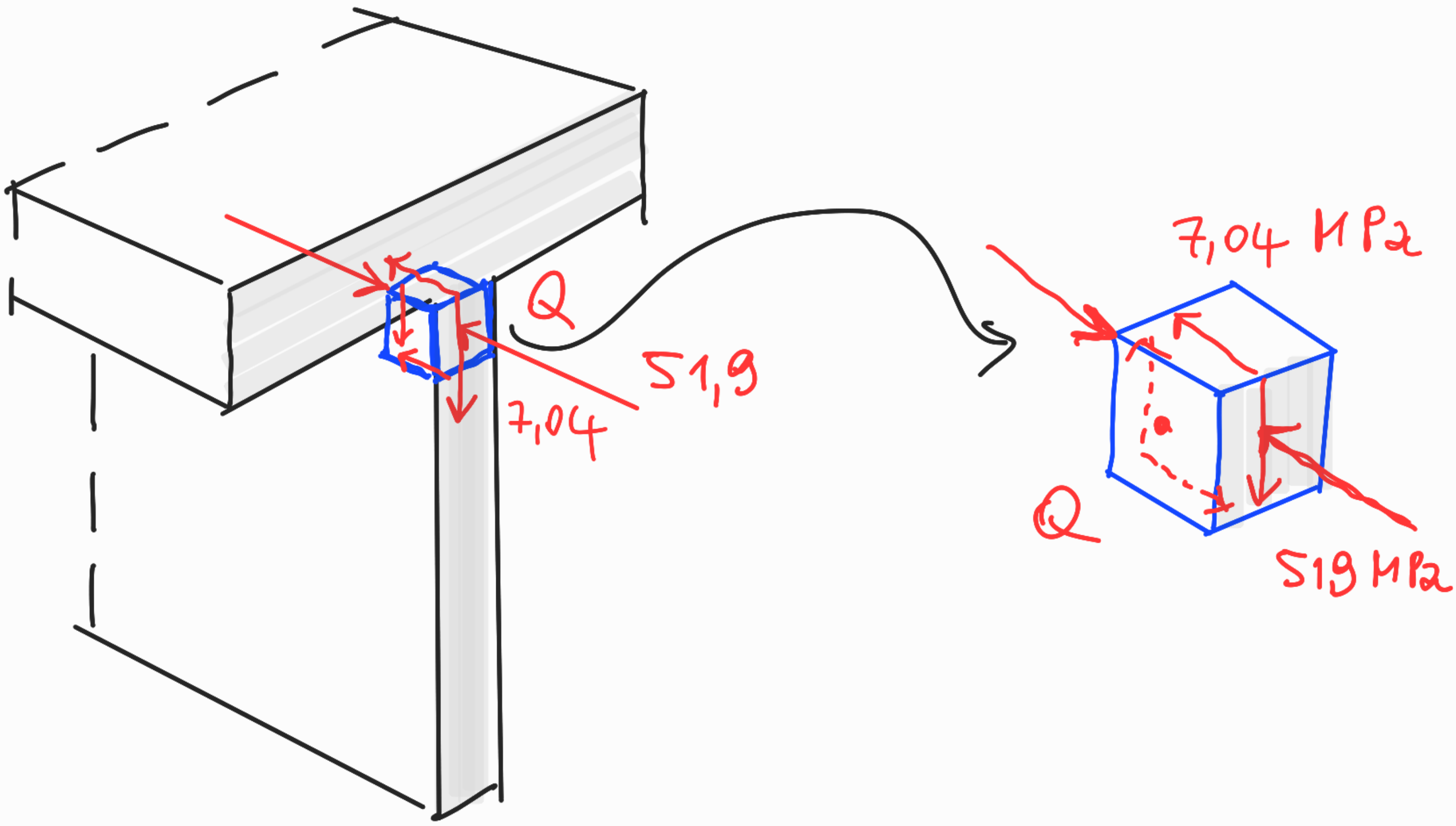
$\tau_{zx}(Q) = 7{,}04\ MPa$

$$\tau_{zy}(\eta) = -T_y \frac{\left[S_x^{(1)} + \eta \cdot 0{,}01 \cdot \left(\frac{\eta}{2} - 0{,}08\right)\right]}{1{,}33 \cdot 10^{-4} \cdot 0{,}01}$$

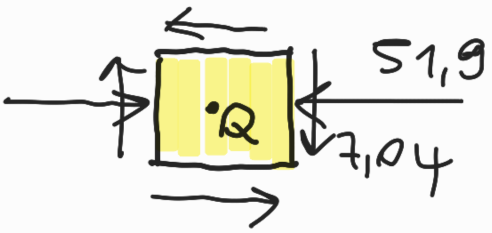


Distribuzione delle tensioni tangenziali nella sezione

- Verifica di resistenza nel punto Q della sez S e cerchio di Mohr



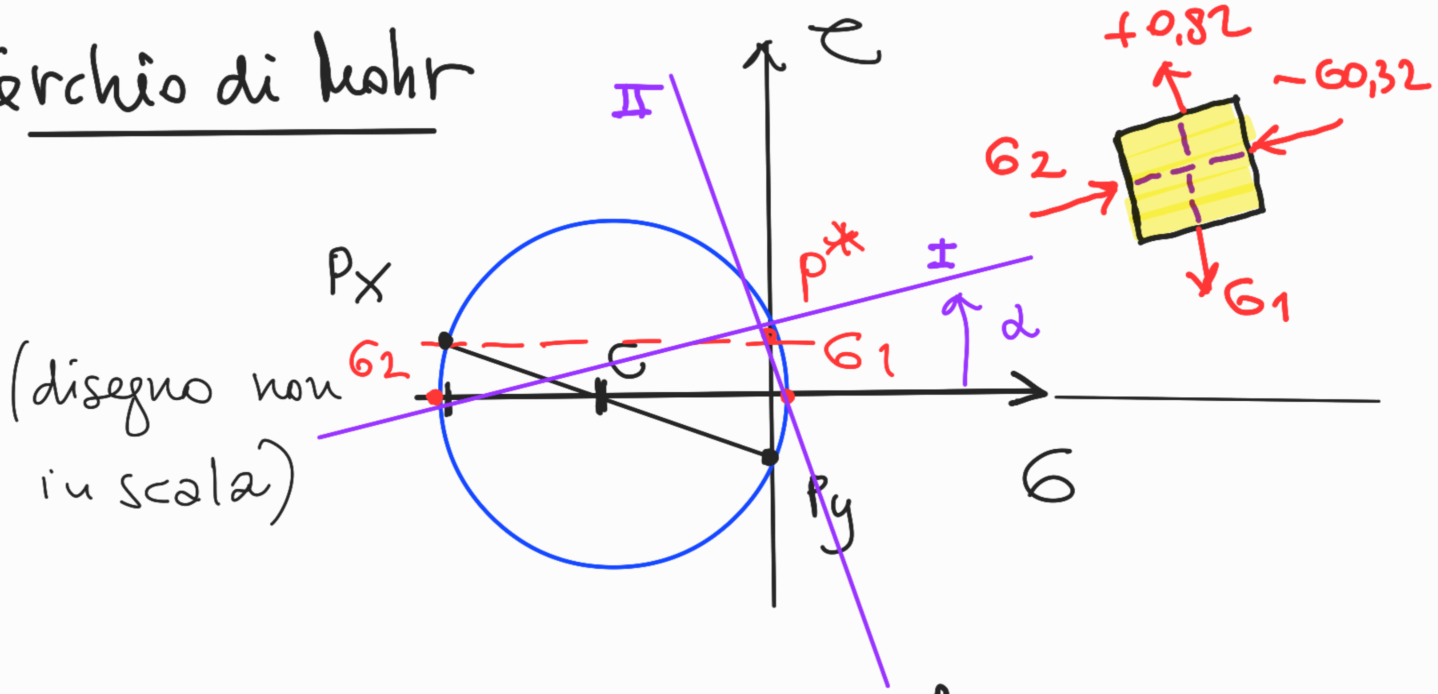
Punto Q (MPa) stato tensionale



$$P_x = (-51{,}9 \ ; 7{,}04)$$

$$P_y = (0 \ ; -7{,}04)$$

Cerchio di Mohr



$$x_c = \frac{-59{,}5 + 0}{2} = -29{,}75 \text{ MPa}$$

$$r = 30{,}57 \text{ MPa}$$

$$\begin{cases} \sigma_1 = x_c + r = 0{,}82 \text{ MPa} \\ \sigma_2 = x_c - r = -60{,}32 \text{ MPa} \end{cases}$$ Tensioni principali

$$\alpha = \frac{1}{2} \operatorname{arctg} \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = 6{,}66^\circ$$ antiorario

- Verifica di resistenza nel punto Q

Criterio di Tresca

$$\sigma_{eq\,TR} = \sqrt{\sigma^2 + 4\tau^2}$$

$$\sigma_{eqTR} = \sqrt{59{,}5^2 + 4 \cdot 7{,}04^2} = 61{,}14 \text{ MPa}$$

$$\sigma_{eqTR} = 61{,}14 \text{ MPa} < 280 \text{ MPa}$$

verificato
