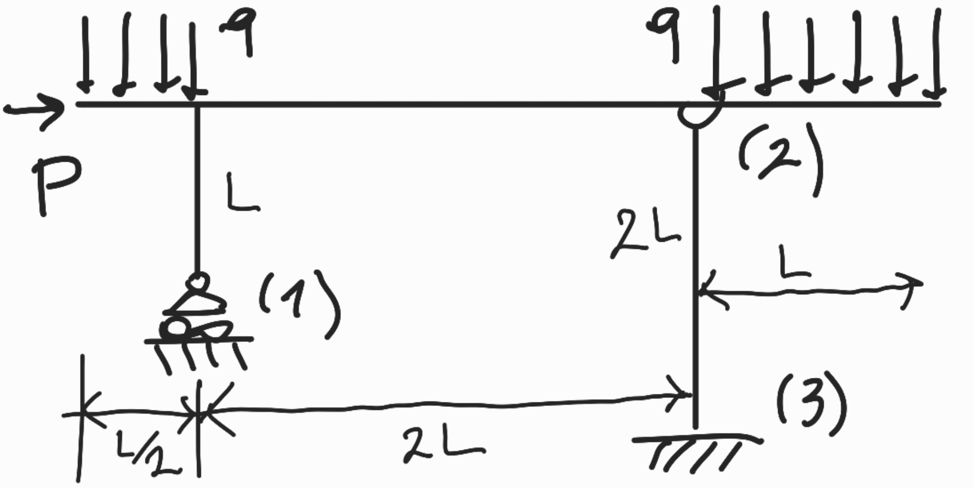
Soluz. Compito Fondam. di Scienza delle Costruz. 15.2.2023

1) • Verifica che la struttura è isostatica

$n_{aste} = 2$ $v = 3+2+1 = 6$

$g = 2 \cdot 3 = 6$ → isostatica

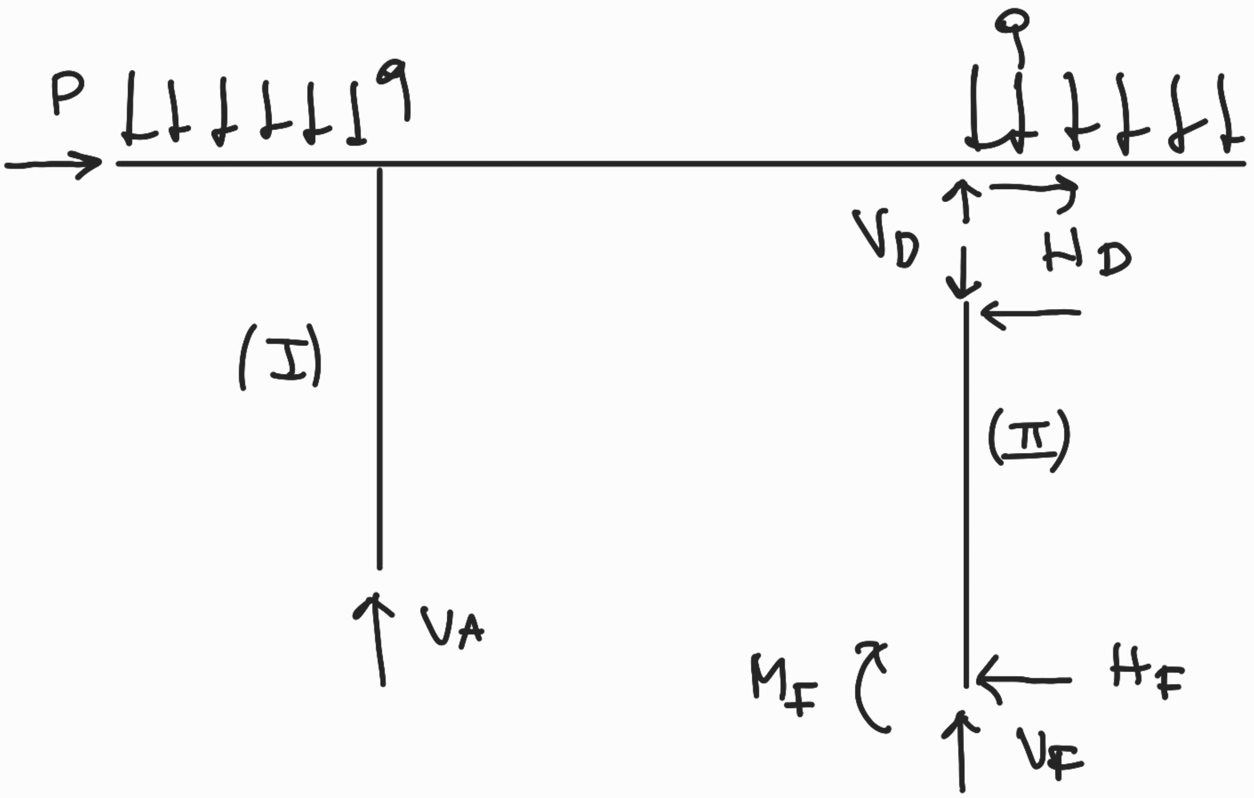


$L = 4$ m

$q = 15000$ N/m

$P = 30'000$ N

• Calcolo delle reazioni vincolari



Equilibri delle aste:

$$\mathrm{I}\begin{cases} \xrightarrow{+} \quad +P + H_D = 0 \\ \uparrow \quad V_A - q\frac{3}{2}L + V_D = 0 \\ \curvearrowleft_D : V_A \cdot 2L - q\frac{L}{2} \cdot \frac{9L}{4} + q\frac{L^2}{2} = 0 \end{cases}$$

$H_D = -P = -30000$ N

$V_D = \frac{3}{2}qL - V_A = 71250$ N

$V_A = 18750$ N

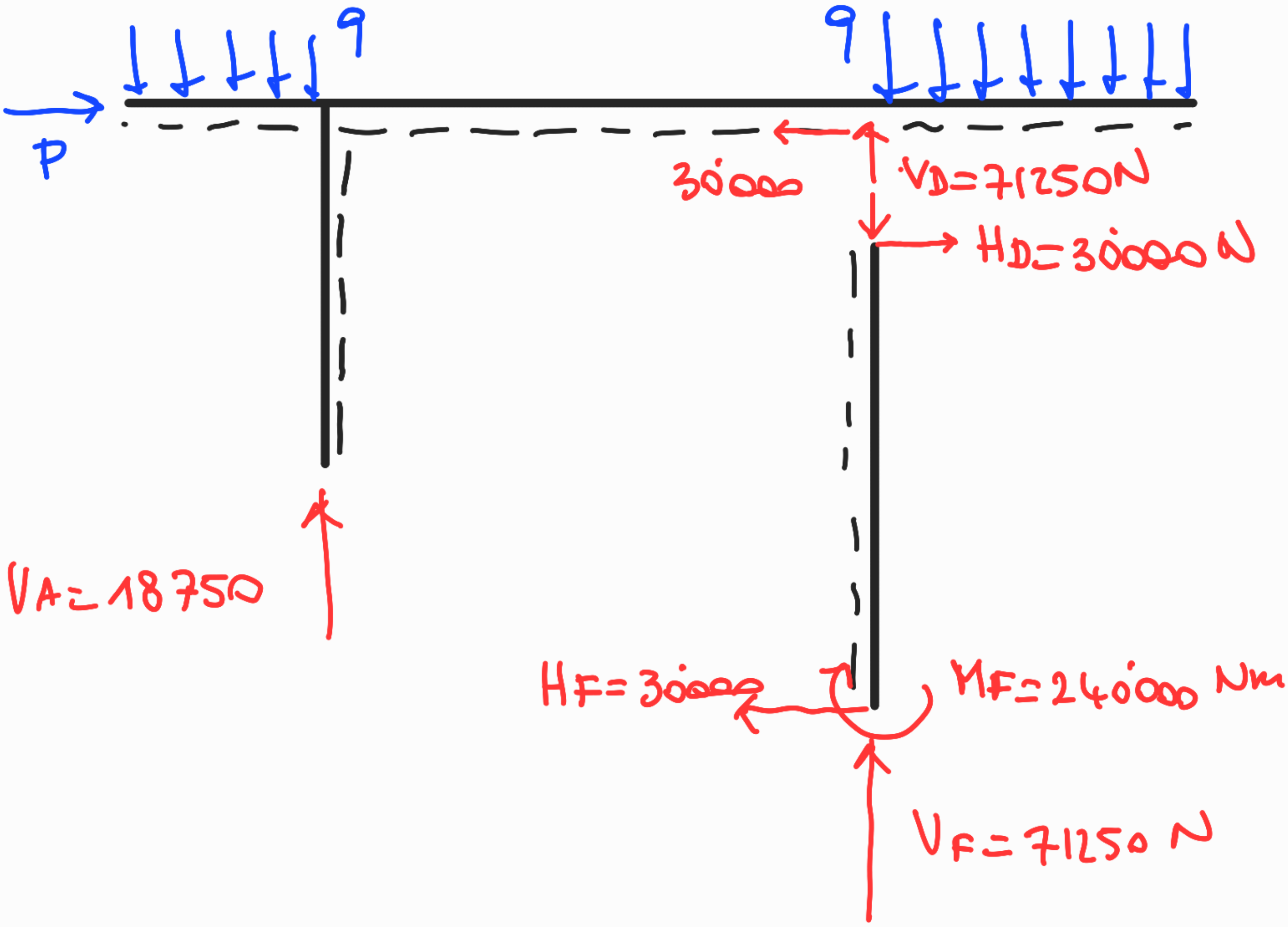
$$\mathrm{II}\begin{cases} \xrightarrow{+} \quad -H_D - H_F = 0 \\ \uparrow \quad V_F - V_D = 0 \\ \curvearrowleft_F : M_F - H_D \cdot 2L = 0 \end{cases}$$

$H_F = -H_D = 30000$ N

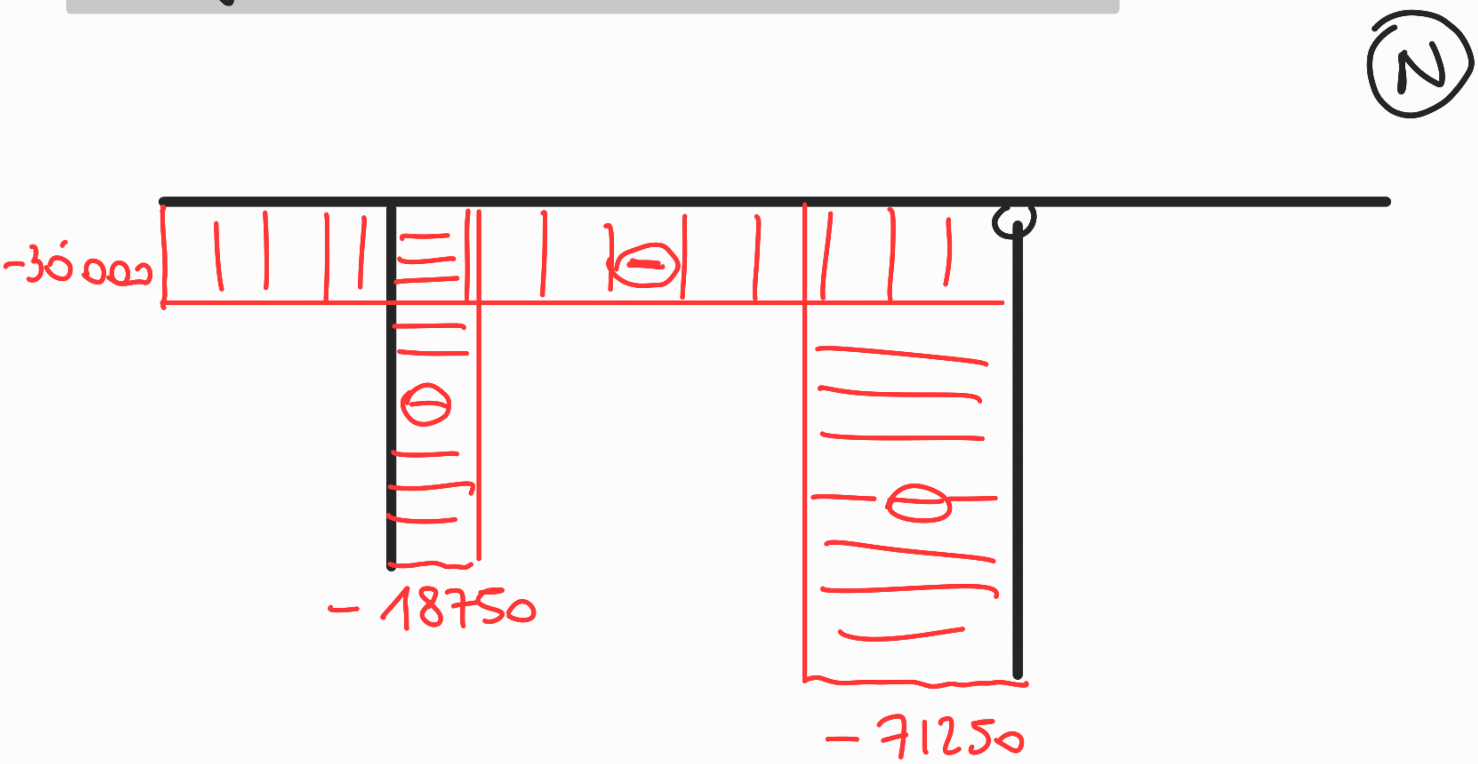
$V_F = V_D = 71250$ N

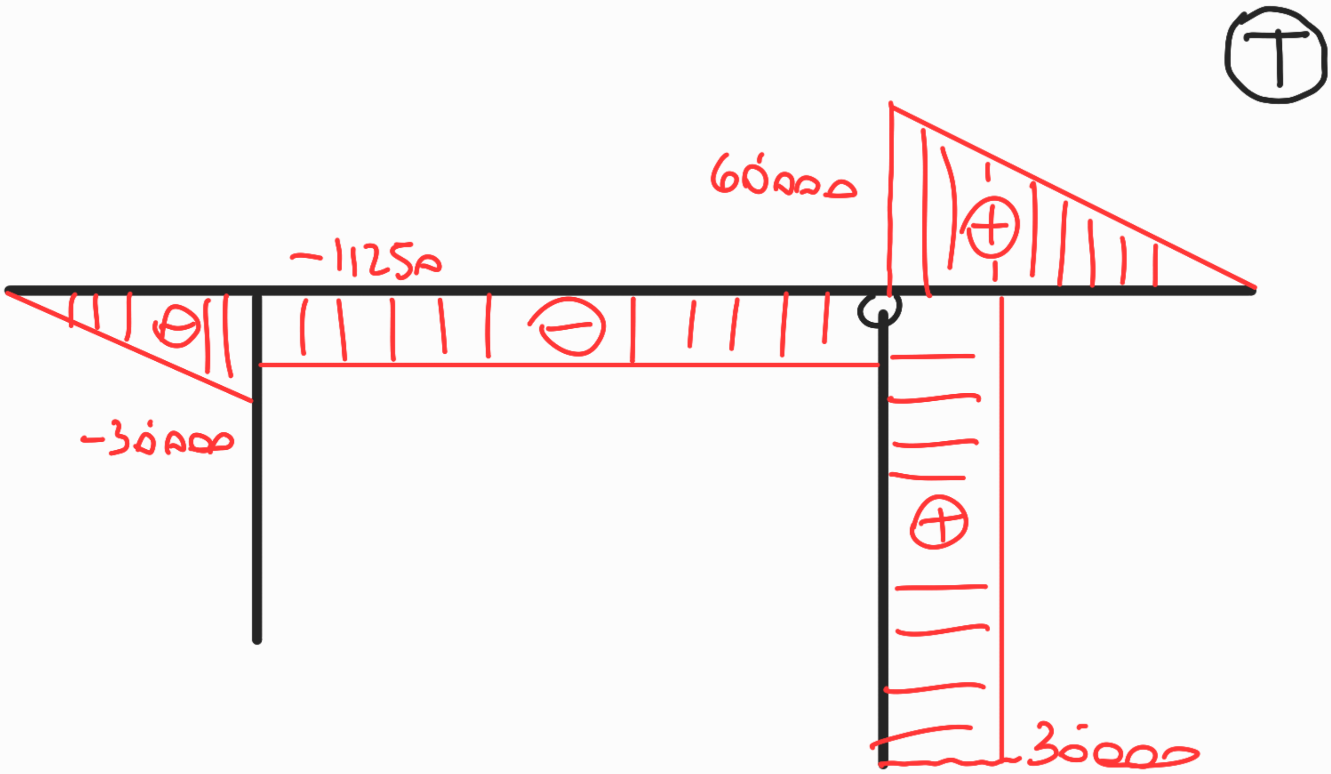
$M_F = H_D \cdot 2L = 240'000$ Nm

Schema finale delle forze attive e reattive

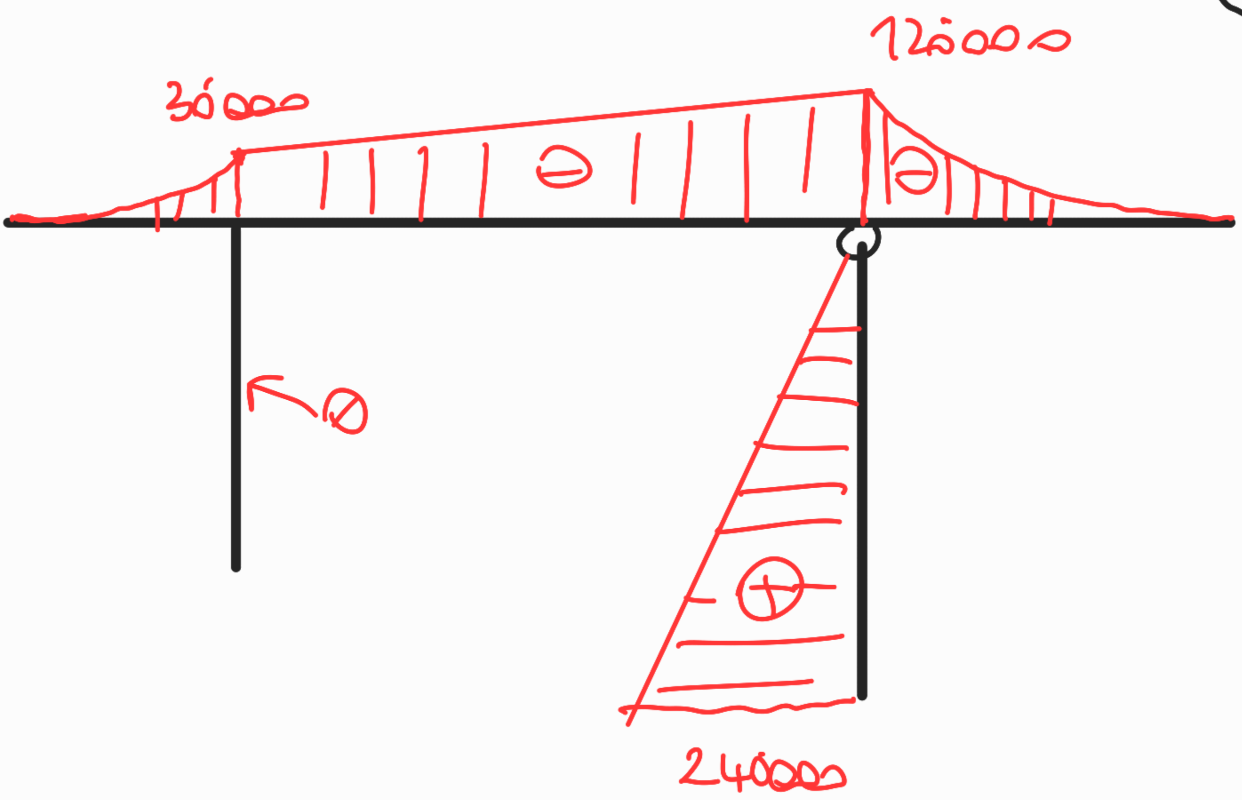


• Diagrammi delle azioni interne



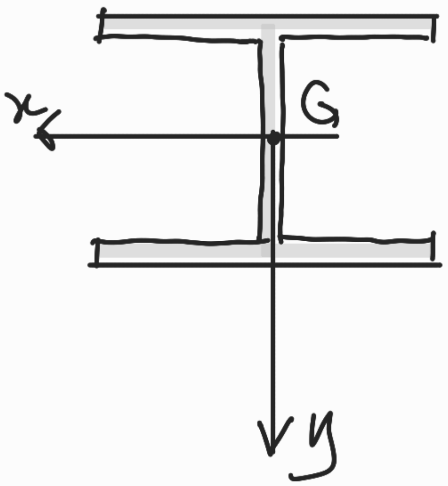


M



2)

- Caratteristiche geometriche della sezione



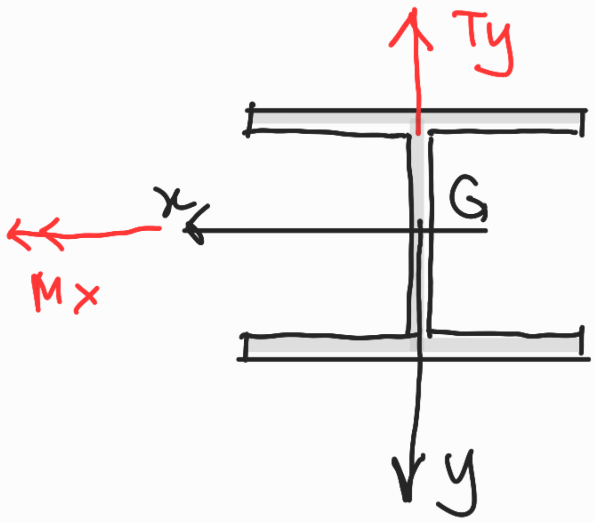
$$A = 6(0{,}2 \cdot 0{,}015) = 0{,}018\ m^2$$

$$I_x = \frac{0{,}015 \cdot 0{,}4^3}{12} + 2 \cdot 0{,}4 \cdot 0{,}015 \cdot 0{,}2^2 =$$

$$= 5{,}6 \cdot 10^{-4}\ m^4$$

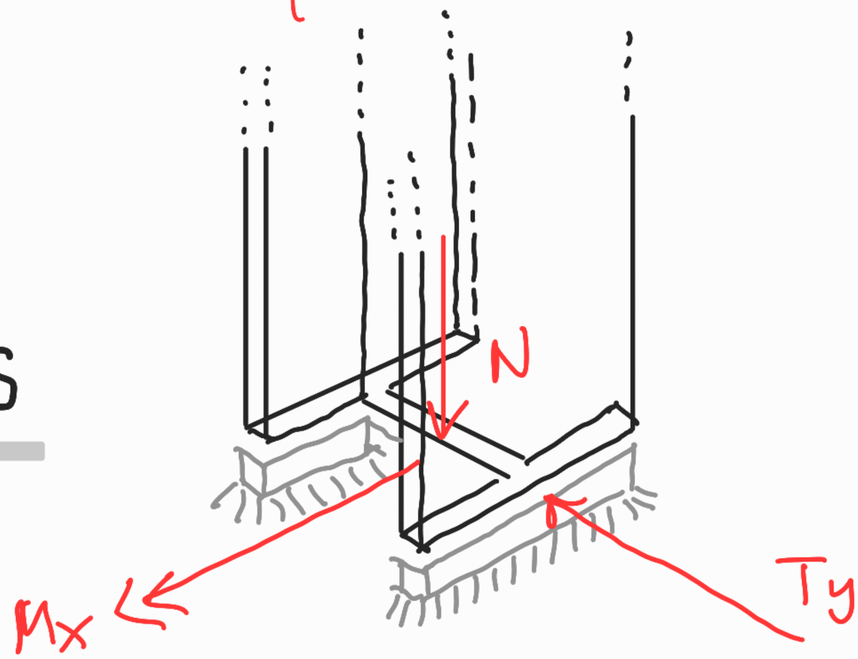
Il momento di inerzia I_y non è necessario.

• Calcolo delle Tensioni nelle sez. S

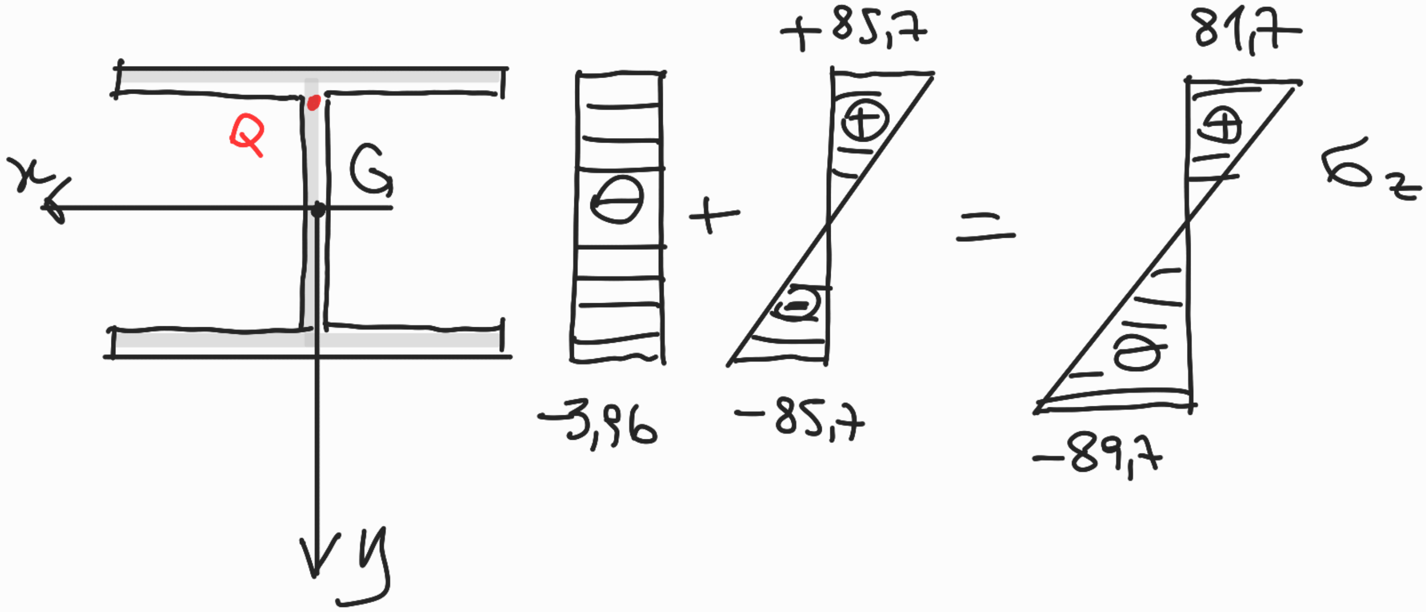


$$\begin{cases} N = -71250 \ N \\ T = 30'000 \ N \\ M_x = 240'000 \ Nm \end{cases}$$

Sez. S-S



• Sforzo normale eccentrico



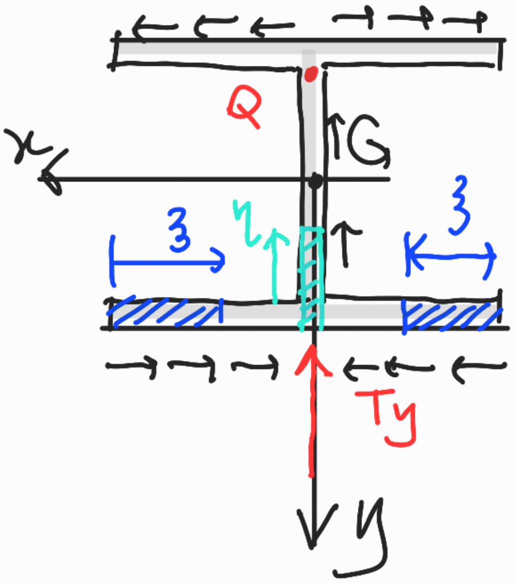
$$\sigma_z(y) = \frac{N}{A} \pm \frac{M_x}{I_x} \cdot y$$

$$= \frac{-71250}{0{,}018} \pm \frac{240000}{5.6 \cdot 10^{-4}} \cdot y =$$

$$= -3{,}96 \cdot 10^{6} \pm 85{,}7 \cdot 10^{6} = \begin{cases} -89{,}7 \text{ MPa} \\ +81{,}7 \text{ MPa} \end{cases}$$

$$\sigma_z(Q) = -95{,}5 \text{ MPa}$$

• Taglio (f. di Jouravski) $\tau_{zs} = -\frac{T_y \, S_x^*}{I_x \cdot b}$



$$\tau_{zx}(\zeta) = -\frac{T_y \cdot 2 \zeta \, t \cdot a}{I_x \cdot 2 \cdot t}$$

$$\tau_{zx}(\zeta = 0{,}2) = \frac{30000 \, (0{,}2 \cdot 0{,}015 \cdot 0{,}2) \cdot 2}{5.6 \cdot 10^{-4} \cdot 2 \cdot 0{,}015} = 2{,}14 \cdot 10^{6} \text{ Pa}$$

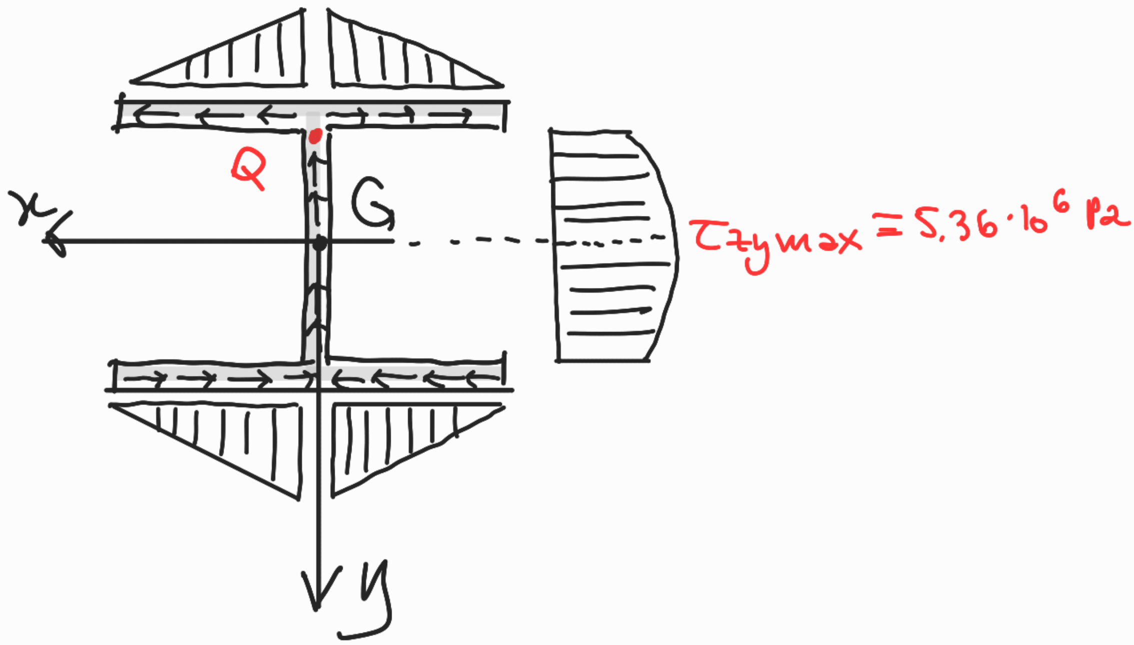
uscente

$$\tau_{zy}(\eta) = -\frac{T_y \left[2 \cdot 0.2 \cdot 0{,}015 \cdot 0{,}2 + \eta \cdot 0{,}015 \cdot \left(a - \frac{\eta}{2} \right) \right]}{5.6 \cdot 10^{-4} \cdot 0{,}015}$$

$$\tau_{zy}(y=0{,}2) = \frac{30000 \cdot 1{,}5 \cdot 10^{-3}}{5.6 \cdot 10^{-4} \cdot 0{,}015} = 5{,}36 \cdot 10^{6} \text{ Pa}$$

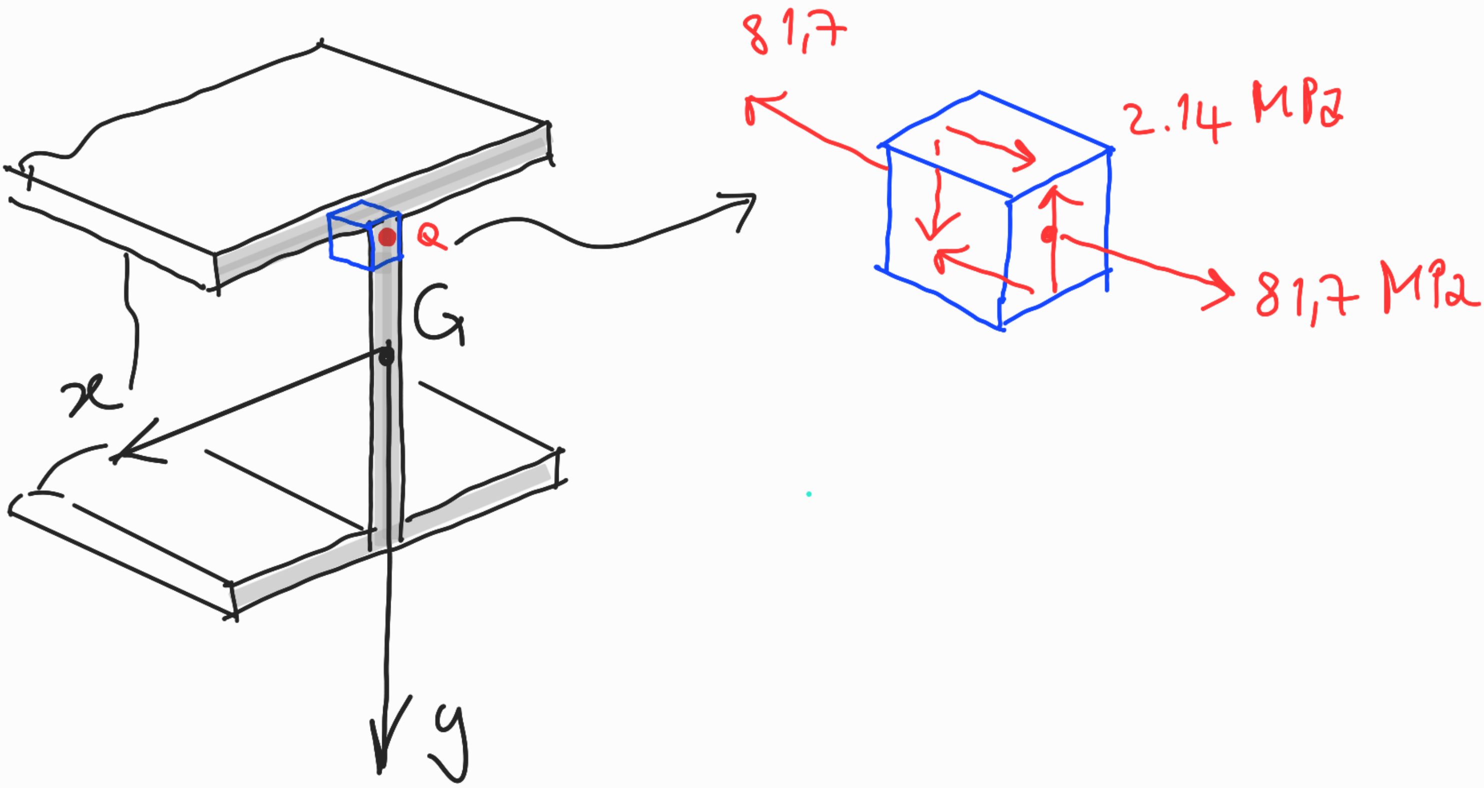
$\tau_{zy\,max}$

$\tau_{zx}(Q) = 2{,}14 \cdot 10^{6}$ Pa

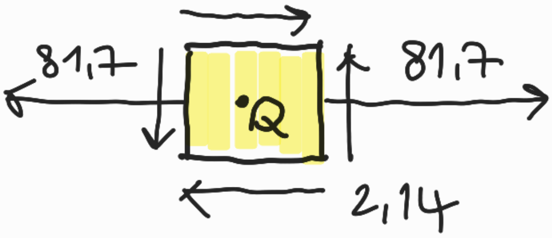


5

- Verifica di resistenza nel punto Q della sez S e cerchio di Mohr

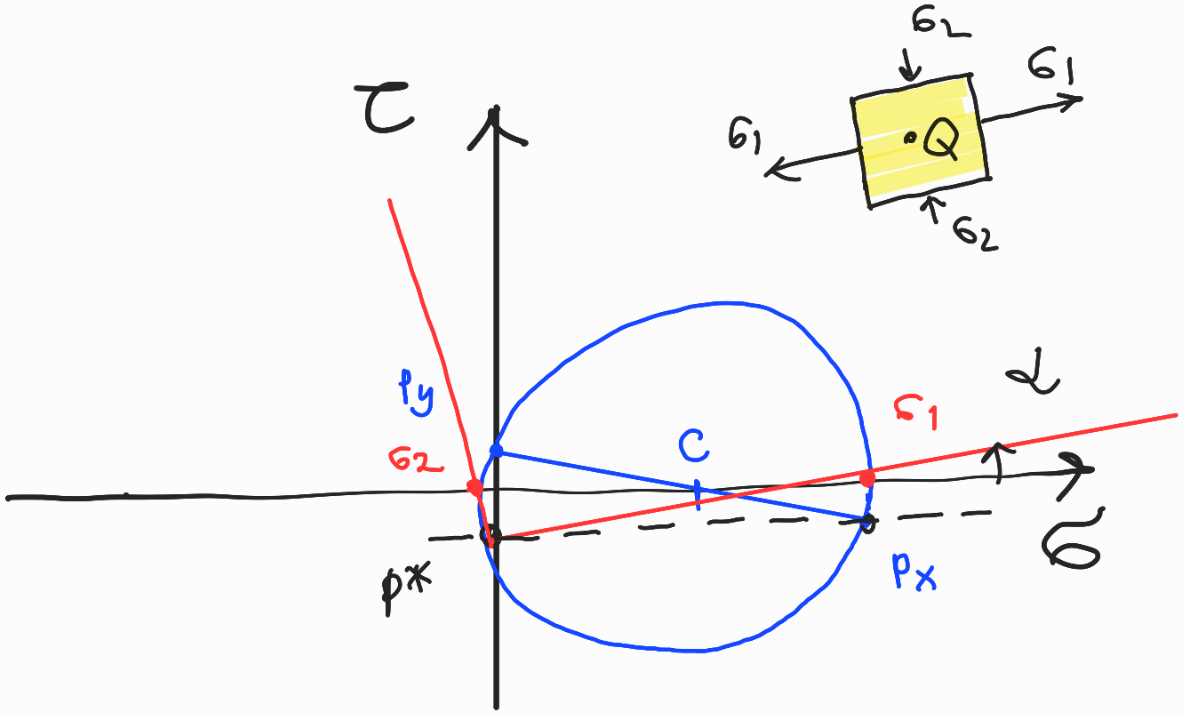


Punto Q (MPa)



$P_x = (81{,}7 \; ; -2{,}14)$

$P_y = (0 \; ; 2{,}14)$



$x_c = +40{,}85$ MPa $r = 40{,}90$ MPa

$$\begin{cases} \sigma_1 = x_c + r = 81{,}75 \text{ MPa} \\ \sigma_2 = x_c - r = -0{,}05 \text{ MPa} \end{cases}$$

$$\alpha = \frac{1}{2} \operatorname{arctg} \frac{\tau_{zy}}{\frac{\sigma_x - \sigma_y}{2}} = +1{,}5^\circ$$ antiorario

- Verifica di resistenza nel punto Q

Criterio di Von Mises

$$\sigma_{ep.VM} = \sqrt{\sigma^2 + 3\tau^2}$$

$$\sigma_{eq\,VM} = \sqrt{81{,}7^2 + 3 \cdot 2{,}14^2} = 81{,}78\ \text{MPa} < 160\ \text{MPa}$$

Verificato
