ADVANCED FINITE ELEMENT ANALYSIS

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1. Exercise

Illustrate the Newton-Rapson algorithm for the solution of a nonlinear problem modelled by using finite elements. Assume only the mechanical nonlinearity (i.e. small strains and displacements). Give a short description of the modified NR algorithm.

2. Exercise

The large deformation of a body is described by uniform extension corresponding to the stretches $\lambda_1 = 1.0, \lambda_2 = 0.5, \lambda_3 = 0.5$. Determine the deformation gradient tensor $\mathbf{F}$, the Green-Lagrange strain tensor $\mathbf{E}$, the engineering strain $\boldsymbol{\varepsilon}$ and the change in volume of the material.
Suppose that the material is of the St.Venant-Kirchhoff type (i.e. $\mathbf{S} = \mathbf{C} \cdot \mathbf{E} = \lambda \cdot \mathrm{tr}\mathbf{E}\,\mathbf{I} + 2\mu\mathbf{E}$). Determine the 2nd Piola-Kirchhoff stress tensor $\mathbf{S}$ (referred to the undeformed configuration) and the true Cauchy stress $\boldsymbol{\sigma}$ (referred to the deformed configuration) by adopting the following elastic constants $\lambda = 1.15 MPa, \mu = 0.77 MPa$.

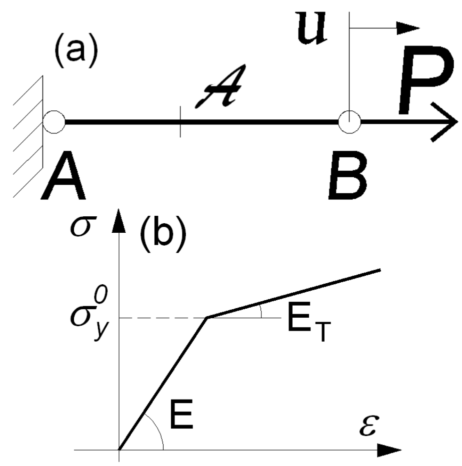
3. Exercise

A bar element, restrained as shown in the figure, is loaded by the axial force P (fig. a). The element has an initial length of 1.0 m and is made of an elastic-plastic material whose stress-strain law is represented by a bilinear curve (see fig. b) with isotropic hardening.

The structure can be modelled through a single truss finite element characterized by the only dof u. The cross section area of the bar is $\mathcal{A} = 5\,\mathrm{cm}^2$, while the load history applied to the element is as follows: $P(t_0) = P_0 = 0,\ P(t_1) = P_1 = 200kN,\ P(t_2) = P_2 = -300kN$.

Determine the plastic modulus H and the history of the resulting tip displacement $u(t)$ of the bar by evaluating first the stress-strain history by using the return mapping algorithm to make the stress-strain point to lie always on the $\sigma - \varepsilon$ curve of the material.

$$E = 2E+11Pa,\ E_T = 5E+10Pa,\ \sigma_y^0 = 3E+8Pa$$



4. Exercise

Discuss the three main modes of deformation of a shell by answering the following questions:

- Provide their general expression in terms of basis vectors and their derivatives.
- Use a schematic to explain the physical meaning of each one of them.
- Based on rigorous mathematical arguments, explain why, in general, bending and stretch are coupled
- Explain the typical simplification made for thin shells.